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E. J. STONE, M.A., F.R.S., Vice-President, in the Chair.

Lord Edward Spencer Churchill, Castlemead, Windsor, was balloted for and duly elected a Fellow of the Society.

The following candidates were proposed as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Bertram Bennett, B.A., Paignton, South Devon (proposed by Edwin Dunkin) ;

Charles Bright, F.R.G.S., Assoc.M.Inst.C.E., M.I.E.E., Telegraph Works, Silvertown, Essex (proposed by the Earl of Crawford) ;

Charles Burckhalter, Assistant, Chabot Observatory, Oakland, California, 962 Chester Street, Oakland (proposed by W. M. Pierson) ;

Arthur Hilton Molesworth, B.A., Barrister-at-law, 15 Park Lane, W. (proposed by A. S. Herschel) ;

R. A. Sampson, B.A., Isaac Newton Student, St. John's College, Cambridge (proposed by A. M. W. Downing) ;

Charles Daulman Webb, B.A., B.Sc., Master in King's College School, 112 Adelaide Road, South Hampstead, N.W. (proposed by H. P. Hollis).

Sixty-five presents were announced as having been received since the last meeting, including, amongst others :—

Opere di Galileo, Edizione nazionale, vol. ii., presented by the Italian Government; Vienna Observatory, Annalen, Band VII., presented by the Observatory ; Munich Observatory, Neue Annalen, Band II., presented by the Observatory ; Paris Ob-

servatory, Catalogue et Positions observées des Étoiles, vi^h à xii^h, presented by the Observatory; Philosophical Transactions of the Royal Society, vol. i., presented by C. L. Prince; two original negatives of the Moon, presented by the Lick Observatory; Leybourn, W., the Art of Dialling, 1682, Munster, S., Rudimenta Mathematica, 1551, and Sturmy, S., The Mariner's Magazine, 1669, presented by William Schooling.

On the Verification of the Expressions given in Delaunay's Lunar Theory by a Direct Differentiation and Substitution in the Differential Equations. By E. J. Stone, M.A., F.R.S.

In the *Monthly Notices* for November, Professor Cayley has shown by a direct differentiation of the expressions v, u, and $\frac{1}{r}$, given by Delaunay, that they satisfy the differential equations employed to the fourth order of the small fractions $\frac{n'}{n}$ or m, and for terms involving only e; and, if I understand the paper rightly, it is suggested that this method might be applied with success to test the general accuracy of Delaunay's expressions.

The general integration of the three simultaneous differential equations has been carried by Delaunay to the seventh order of the small fractions involved, whilst for certain terms, in which this degree of approximation was found insufficient, special investigations have been undertaken for the determination of the coefficients to much higher orders than the seventh; and the necessary transformations have been effected to give the expressions for v and u complete to the seventh order. But, because the inverse radius vector is ultimately multiplied by a comparatively small linear quantity, the radius of the Earth, and is compared directly with observation under the form of the Lunar Parallax, it has been thought sufficient to give the algebraical expression for $\frac{1}{r}$ only to the fifth order of small quantities; and the coordinates v, u, and $\frac{1}{r}$, as given by Delaunay, are therefore only complete to the fifth order. There would, however, be no very serious difficulty in effecting the necessary transformations and obtaining the expression for $\frac{1}{r}$ to the seventh order of small quantities, and this would render the expressions for v, u, and $\frac{1}{r}$, if accurate, complete to the seventh order. But the labour of extending these expressions generally to the high order of small quantities which has been found necessary for some terms would

be enormous; and this work is hardly likely to be undertaken and carried out. But unless the expressions for the coordinates v, u, and $\frac{1}{r}$ are complete to some definite order, p, it will, I believe, be found impossible to verify the accuracy of the work of integration in series by the inverse process of a direct differentiation of these expressions for v, u, and $\frac{1}{r}$, and the substitution of the results in the differential equations in the manner which has been successfully adopted by Professor Cayley for terms of order e. and to m^4; and if sensible errors exist in Delaunay's expressions they are far more likely to be found in terms of an order higher than the seventh, at which the general integration ends, than amongst those of a lower order.

But, even if the complete integration of the equations employed by Delaunay had been effected, it would be quite impossible to make the functions v, u, and $\frac{1}{r}$ accurately represent the geocentric coordinates of the Moon's centre of gravity by assigning any definite numerical values to the constants a e, γ, τ, g, and h, introduced in the integrations, and of the constant μ, or

$$n = \sqrt{\frac{\mu}{a^3}},$$

because the equations integrated are not the differential equations of motion. And it will be impossible to pass from the integrals v, u, and $\frac{1}{r}$ to the required integrals of the differential equations of motion,

$$v + \delta v,\ u + \delta u,\ \frac{1}{r} + \delta\frac{1}{r}$$

until at least the corrections which the expressions v', u', $\frac{1}{r'}$ adopted for the geocentric coordinates of the Sun are rendered definite, and if the final corrections δv, δu, and $\delta\frac{1}{r}$ are found on the assumption that $\delta v'$ contains no terms of the form

$$\delta(l' + n't)$$

or

$$\delta l' + \delta n'.t + n'\delta t,$$

then the resulting functions,

$$v + \delta v,\ u + \delta u,\ \frac{1}{r} + \delta\frac{1}{r},$$

will, if complete, be the expressions for the geocentric coordi-

nates of the Moon when t is found from observation, subject to the conditions indicated, that $\delta v'$ contains no terms of the form

$$\delta(l' + n't)$$

or

$$\delta l' + \delta n' t + n' . \delta t,$$

but not otherwise.

If such corrections as $\delta l' + \delta n' . t + n' . \delta t$ exist and are sensible, the effects of such terms on the integrals $\delta v . \delta u$. and $\delta . \frac{1}{r}$ must be investigated, and included in the expressions for $v + \delta v . u + \delta u \frac{1}{r} + \delta \frac{1}{r}$ before the theoretical results are compared with observation for the determination of the numerical values of the constants involved in their expression. But if $\delta l' + \delta n' . t + n' . \delta t$ is put equal to zero, then the variable t, or the time, must be found subject to the same conditions; and this requires, in order that $\delta t = 0$, or the time, t, be found correctly, that $\delta l' = 0$ and $\delta n' = 0$, or that the epoch from which t is measured, and the unit in which it is expressed, shall be such that the adopted values l' and n' are exact.

If the integrals of the differential equations of motion are correctly found in terms of the variable t, which renders $\delta l' = 0$, $\delta n' = 0$, or the adopted values of l' and n' exact, and we employ in making the comparisons between theory and observation a value $t + \delta t$ instead of t, the effects of the errors, δt, will be thrown upon the determinations of the constants; and this will be true however the variable $t + \delta t$ may be found from observation, provided it differs from the time, t, on the required scale of time measurements; and if δt is of the form P sin $(pt + q)$, the determinations of the epoch-constants and mean motions will appear liable to periodical changes which cannot be accounted for, quantitatively, by direct theoretical investigations of the motions of the planets and of the Moon, based on the usual assumption that $\delta(l' + n't) = 0$. There will appear in such cases a necessity for empirical long inequalities to secure an agreement between the tabular results and those of observation. But such empirical corrections should, I believe, be most carefully avoided, and attention directed to the theoretical inequalities in solar theory to avoid errors, δt, and to the completion of the theories of the planets and Moon on the present assumption that $\delta(l' + n't) = 0$. It may be mentioned that the right ascensions of the stars are found subject to the same condition, and that from them the right ascensions of the meridians are interpolated by the aid of clocks, and rendered definite by direct references to the Sun's positions at the meridian passages.

On the Determination of a certain Class of Inequalities in the Moon's Motion. By Ernest W. Brown, B.A.

1. The two principal methods of treating the Lunar Theory—viz. (1) the *general* method, in which we obtain a general approximate solution, applying to any single Moon whose present orbit lies within given limits, in terms of certain constants, the coordinates of position and the time; and (2) the *special* method, in which we use the numerical values of those constants at the outset for any given Moon, and obtain a solution involving the coordinates of position and the time only—present each of them certain advantages. In the first case, we have the theory, worked out to a certain degree of accuracy, immediately applicable to any single Moon in our solar system, and therefore arranged in such a way that any small change which improved data may involve in the values of the constants can be made easily without requiring us to go over the whole of the work again. On the other hand, the number of terms which have been found necessary to secure a degree of accuracy commensurate with that of observation is very large, and it becomes a task of great labour to obtain them with any degree of certainty. In the second case, when we use numerical values from the start, the portions arising from the expansion of the functions in ascending powers of the constants involved, which portions in the first case would have to be neglected, are no longer left out of account. A great increase of accuracy naturally results. But this method is unsatisfactory in some respects. Having started with certain values of the constants for any particular Moon, we are bound to keep to those values; and also if any numerical mistake be made, it is not very easily traceable. The calculation is applicable only to the particular Moon selected. Any method, therefore, of finding the path of the Moon which will give at the same time numerical and algebraical results with equal accuracy would be of value.

Mr. G. W. Hill, in the *American Journal of Mathematics*, vol. i.,* has shown how to separate out those inequalities dependent *only* on the ratio of the mean motions of the Sun and Moon, and to obtain their coefficients both numerically and as an algebraic function in ascending powers of this ratio. He shows also that his method is not only susceptible of a very high degree of accuracy with comparatively little trouble, but that any higher degree of accuracy can be obtained without the necessity of going over a large part of the ground again. His method for this class of inequalities is as follows:—

2. Take as origin the centre of the Earth considered as a sphere, and refer the motion to rectangular axes revolving round the origin with the mean angular velocity n' of the Sun round the Earth. If we neglect all the differences from purely elliptic

* *Researches in the Lunar Theory*, pp. 5, 129, 245

motion, but those dependent on the mean motions of the Sun and Moon, the Sun will describe a circle about the Earth with constant angular velocity n', and we may therefore assume the moving axis of x to pass through the Sun. Under these circumstances, the equations of motions of the Moon become

$$\left.\begin{aligned} \frac{d^2x}{dt^2} - 2n'\frac{dy}{dt} + \left(\frac{\mu}{r^3} - 3n'^2\right)x &= 0 \\ \frac{d^2y}{dt^2} + 2n'\frac{dx}{dt} + \frac{\mu}{r^3}y &= 0 \end{aligned}\right\} \quad \text{I.}$$

These equations will give us all inequalities from purely circular motion, which involve only the eccentricity of the Moon and the mean motions of the Sun and Moon. By taking certain "particular integrals" of these equations, the inequalities dependent only on the mean motions can be separated out and determined. This particular solution is given by

$$x = \Sigma a_{2i} \cos (2i+1)(n-n')(t-t_0)$$
$$y = \Sigma a_{2i} \sin (2i+1)(n-n')(t-t_0),$$

when i takes integral values positive and negative, and a_{2i} is a function only of the ratio of the mean motions of the order m^{2i} at least. The quantities a_{2i} are then determined by substituting this particular solution in the equations of motion. Certain transformations are first made in order to reduce the equations (I) to a convenient form.

3. In most previous methods, and especially in Delaunay's, the coefficients of this class of inequalities are expressed directly in ascending powers of $n'/n = m$; but in Mr. Hill's method of preparing the equations for solution the function of m which appears most naturally for use is $n'/(n-n')$. Let

$$m_1 = \frac{n'}{n-n'} = \frac{m}{1-m};$$

we then have

$$m = \frac{m_1}{1+m_1}.$$

At first sight m_1 would seem to be a less suitable quantity than m, since m_1 is greater than m, and consequently a greater number of terms would seem to be required to obtain a given degree of accuracy. But it is shown that the numerical multipliers of the higher powers of m_1 are much smaller than those of the corresponding series in m; in other words, the series for the coefficients expressed in ascending powers of m_1 are so much more convergent than the same series expressed in powers of m, as to quite overbalance the disadvantage of m_1 being slightly greater than m. Further, he shows that by using a certain function of m_1 *obtained theoretically*, for expansion, the series can be ren-

dered still more convergent. This function of m_1 is μ, where μ is given by

$$\mu = \frac{m_1}{1 - \frac{1}{3}m_1} = \frac{m}{1 - \frac{4}{3}m}.$$

Hence

$$m = \frac{\mu}{1 + \frac{4}{3}\mu}.$$

The series for a_{2i} are given by Mr. Hill, developed in terms of both m_1 and μ.

4. These transformations can be utilised also in another way. The inequalities found as indicated in § (2) are referred to rectangular coordinates. It has been usual to refer the Moon's motion to polar coordinates. We have for this purpose

$$r\cos v = \Sigma a_{2i}\cos 2i\,(n-n')\,(t-t_0)$$
$$r\sin v = \Sigma a_{2i}\sin 2i\,(n-n')\,(t-t_0),$$

where r is the radius vector and v the excess of the true over the mean longitude. From these r and v can be easily found.

The process of finding these coefficients as algebraic expansions in powers is somewhat laborious when, as in the special case of the Variation Inequality, it is necessary to carry the expansions to a high power. But there is no reason why, supposing Delaunay's expressions in longitude and parallax for these coefficients to be accurate as far as they go, we should not put in them

$$m = \frac{\mu}{1 + \frac{4}{3}\mu},$$

and expand in powers of μ. This transformation gives us smaller numerical multipliers of the higher powers, and consequently greater accuracy when we substitute for μ its numerical value. It is to be noted that if the given series is calculated as far as m^q, the transformed series cannot be carried further than μ^q.

5. As an example, take that part of Delaunay's expression * for the coefficient of the Variation which depends on m. It is

$$\frac{11}{8}m^2 \quad +\frac{59}{12}m^3 \quad +\frac{893}{72}m^4 \quad +\frac{2855}{108}m^5 \quad +\frac{8304449}{165888}m^6$$

$$1586''{\cdot}8883 \quad +424''{\cdot}4474 \quad +80''{\cdot}0906 \quad +12''{\cdot}7689 \quad +1''{\cdot}8087$$

$$+\frac{102859909}{1244160}m^7 \quad +\frac{7596606727}{74649600}m^8 \quad -\frac{805141816}{1119744000}m^9 \quad . \quad . \quad . \quad (\alpha)$$

$$+0''{\cdot}2234 \quad +0''{\cdot}0206 \quad -0''{\cdot}0001$$

where the coefficients expressed in seconds are written underneath. Putting *in this series*

$$m = \mu/(1 + \tfrac{4}{3}\mu),$$

* Delaunay, *Mémoires de l'Académie des Sciences*, tome xxix. p. 815.

and expanding in powers of μ, it becomes

$$\begin{array}{lllll} \frac{11}{8}\mu^2 & +\frac{5}{4}\mu^3 & +\frac{5}{72}\mu^4 & -\frac{11}{36}\mu^5 & -\frac{82111}{165888}\mu^6 \\ 1957''{\cdot}9686 & +147''{\cdot}8944 & +0''{\cdot}6827 & -0''{\cdot}2496 & -0''{\cdot}0336 \end{array}$$

$$\begin{array}{lll} -\frac{350399}{138240}\mu^7 & -\frac{233559113}{74649600}\mu^8 & -\frac{10961275281}{1119744000}\mu^9 \\ -0''{\cdot}0143 & -0''{\cdot}0015 & -0''{\cdot}0004 \end{array} \quad \ldots \quad (\beta)$$

The values of the coefficients obtained from these expressions are—

$$\left.\begin{array}{lll} \text{From } (\alpha) & \ldots \quad \ldots & \ldots \quad 2106''{\cdot}2478 \\ \text{From } (\beta) & \ldots \quad \ldots & \ldots \quad 2106{\cdot}2463 \\ \text{From Mr. Hill's values} & & \ldots \quad 2106{\cdot}2463 \end{array}\right\} \quad \ldots \ldots \quad (\gamma)$$

The last value has been calculated from the numerical values of the coefficients a_{2i} given in his paper already referred to. The equality of the second and third values—the second obtained from Delaunay's series when expressed in terms of μ, and the third from Mr. Hill's numerical values—is striking. It not only shows us the improvement in accuracy afforded by such transformations, but it gives us also some security for the general correctness of this particular part of Delaunay's expression for the coefficient of the variation.

6. There appears to be a kind of oscillation in the class of series of which (α) or (β) is an example. For instance, in the series (α) the numerical multipliers are positive, and increase gradually until a change of sign occurs between m^8 and m^9, when there is a sudden drop. The numerical values (γ) seem to show that there must be a large increase in the multipliers of the powers after m^9, and that they must have negative signs; so that probably the multipliers of m^{10} and m^{11} are much larger than even those of m^7 or m^8. The series (β) has a sudden drop between μ^3 and μ^4, and then a change of sign occurs, after which the multipliers gradually increase again. I have calculated similar expressions in the coefficients of other inequalities, and generally it appears that when we, by any such substitution as that made above, apparently improve the convergency of the series, the change of sign is brought nearer to the beginning of the series. An ideal to aim at would seem to be, firstly, that the numerical multipliers be made as small as possible; secondly, that there be no sudden increase in them in the later part of the series; and thirdly, that these two conditions should involve that there be no long run of powers with the same sign attached to them. This appears very forcibly in the expressions for the Parallactic Inequality given in § (11) below. It should be stated that as the series are calculated up to some definite power only,

and as the law of progression of the series is not able to be expressed by an algebraical formula, owing to the complicated forms from which they arise, a substitution like that made above *must not be arbitrary*, but must be indicated by theory. An arbitrary substitution, which would seem to make the parts of the series already calculated converge rapidly, may make the unknown part very slowly convergent, and thus introduce unknown errors into the numerical values of the coefficients.

7. It has been mentioned above that the equations (I.) determine all the inequalities dependent on the eccentricity and the mean motion only. As in these equations when transformed for solution m_1 and not m is the quantity which appears, all those series which depend on the eccentricity and mean motions only will probably be rendered more convergent when expressed in terms of m_1. Consequently, in Delaunay's expressions involving e and m only, we are entitled to put $m=m_1/(1+m_1)$, and expand in powers of m_1 up to the same power of m_1 as that to which m was carried; the result of this should be more convergent series and greater accuracy in the coefficients when expressed in seconds. It will be found on examination that nearly all these series have a number of successive powers of m of the same sign with gradually increasing multipliers; and the transformation improves the series in the manner mentioned in § (6).

8. When we examine Delaunay's expression for the part of the coefficient of the Parallactic Inequality dependent on the ratios of the mean distances and mean motions, we notice that the numerical multipliers of the higher powers of m become very large. The multiplier of any one power is between four and seven times that of the previous power; and since m is in the case of our Moon about ·074, the series expressed in seconds converges very slowly. The expression is given below in § (11). In fact, the last term calculated, which is of the ninth order, corresponds to a coefficient of $0''{\cdot}38$; so that some method is necessary for this inequality which will give us either more terms easily, or a new form of expression with greater convergency. I shall give elsewhere the analysis by which the expressions and values below have been obtained, and expose only the method used to work up the coefficients into a more convergent form. No other inequalities but those given are affected thereby.

In obtaining these inequalities dependent on m and the ratio of the mean distances, some terms of the disturbing function have to be added to the left-hand sides of the equations (I.). The constant m_1 is still the function of m which appears naturally for use in expansion. The general method of procedure has been the same as that Mr. Hill used in getting the inequalities dependent on the mean motions only.

9. Let a denote the mean semi-axis major of the Moon's orbit, a' that of the Sun's orbit, v the difference between the true and mean longitudes of the Moon, 2D the argument of the variation

in Delaunay's theory; the parts of the Lunar Inequalities dependent on the mean motions and a/a' only, appear in the following form:—

$$r\cos v = a_0 + (a_1+a_{-1})\cos \mathrm{D} + (a_2+a_{-2})\cos 2\mathrm{D} + \ldots$$
$$r\sin v = \quad (a_1-a_{-1})\sin \mathrm{D} + (a_2-a_{-2})\sin 2\mathrm{D} + \ldots$$

The quantities $a_{\pm p}$ are functions of the kind,

$$\frac{a_{\pm 2p}}{a} = f(m_1) + \left(\frac{a}{a'}\right)^2 \phi(m_1) + \ldots$$
$$\frac{a_{\pm 2p+1}}{a} = \frac{a}{a'}\mathrm{F}(m_1) + \left(\frac{a}{a'}\right)^3 \Phi(m_1) + \ldots$$

where p is a positive integer.

In the functions f for a_{2p}, ${m_1}^{2p}$ is the lowest power of m_1 which occurs; in F the lowest power is also ${m_1}^{2p}$, except when $p=o$, when it is m_1; when $p=o$, f is of the form $1+\alpha {m_1}^2+\beta {m_1}^3+\ldots$, α, β, .. being numbers. Mr. Hill, in his paper referred to, has given the functions f, and his values for them have been used in obtaining the functions F, ϕ, Φ.

When we transform these expressions to polar coordinates, we have, neglecting cubes and higher powers of a/a',

$$v = \mathrm{A}\frac{a}{a'}\sin \mathrm{D} + \left\{\mathrm{B}+\mathrm{B}'\left(\frac{a}{a'}\right)\right\}\sin 2\mathrm{D} + \mathrm{C}\frac{a}{a'}\sin 3\mathrm{D} + \ldots$$

where A, B, B', C . . . are functions of m_1. D is the argument of the Parallactic Inequality, and the first term represents the principal part of it, *i.e.* the part dependent on m_1 and a/a' only. The full expression of this inequality contains terms dependent on the solar and lunar eccentricity and the lunar inclination, but we are not here concerned with such terms, and they do not affect the expressions which follow.

10. The principal part of the coefficient $\mathrm{A}a/a'$ depends chiefly on a_1/a and a_{-1}/a. In obtaining a_1 and a_{-1} as far as the order ${m_1}^5$. a/a', it is possible so to arrange the work that the equations determining them are linear and of the form

$$\frac{a_1}{a_0}\cdot\alpha_1 + \frac{a_{-1}}{a_0}\cdot\beta_1 = \kappa_1\cdot\frac{a}{a'},$$
$$\frac{a_1}{a_0}\cdot\alpha_2 + \frac{a_{-1}}{a_0}\cdot\beta_2 = -\kappa_2\cdot\frac{a}{a'}$$

where α_1 α_2 β_1 β_2 κ_1 κ_2 are functions of m_1 expanded in ascending powers of m_1, and the signs in the equations are so arranged that these functions become positive numerical quantities in the special case of our Moon.

Solving these equations for a_1 and a_{-1} we have

$$\frac{a_1}{a_0} = -\frac{\kappa_1\beta_2+\kappa_2\beta_1}{\beta_1\alpha_2-\alpha_1\beta_2}\cdot\frac{a}{a'} \qquad \frac{a_{-1}}{a_0} = +\frac{\kappa_1\alpha_2+\kappa_2\alpha_1}{\beta_1\alpha_2-\alpha_1\beta_2}\cdot\frac{a}{a'}.$$

The expressions on the left-hand sides of these equations, when for α_1 β_1, etc. are substituted their expansions in powers of m_1, are cumbrous. At the same time, when α_1 β_1, etc. are given their numerical values, we get a very close approximation to the results obtained by the more accurate method of using numerical values from the start. Each of these quantities, when expanded in powers of m_1, converges quickly and no very large numerical multipliers seem to appear. But the multipliers in all of them gradually increase, and consequently, when we express the products κ_1 β_2, κ_2 β_1, etc., in ascending powers of m_1, larger multipliers occur. It seems best, however, for simplicity to expand the numerators and denominators of the expressions for a_1/a_0 and a_{-1}/a_0, and leave the results in the form of fractions which have the same denominator. The loss in accuracy is very small. We have then

$$\frac{a_1}{a_0} = -\frac{\gamma(m_1)}{\tau}\cdot\frac{a}{a'} \qquad \frac{a_{-1}}{a_0} = +\frac{\delta(m_1)}{\tau}\cdot\frac{a}{a'}$$

where γ, δ, τ are functions of m_1 expanded in ascending powers.

11. The terms in longitude with arguments D, 3D, 5D, calculated in this way, become

$$\left[-\frac{\frac{15}{8}m_1 + \frac{9}{4}m_1^2 - \frac{1951}{128}m_1^3 - \frac{41585}{1024}m_1^4 - \frac{2096751}{49152}m_1^5}{1 - 4m_1 - \frac{37}{8}m_1^2 - \frac{17}{6}m_1^3 - \frac{89963}{9216}m_1^4} - \frac{35}{1024}m_1^4 - \frac{1345}{12288}m_1^5 \right] \times \frac{a}{a'}\sin \mathrm{D}$$

$$-124''{\cdot}1493 \quad -12''{\cdot}0447 \quad +6''{\cdot}5970 \quad +1''{\cdot}4218 \quad +0''{\cdot}1207$$

$$-0''{\cdot}0008 \quad -0''{\cdot}0002$$

$$\left[+\frac{15}{32}m_1^2 + \frac{55}{128}m_1^3 - \frac{41}{1536}m_1^4 + \frac{2309}{9216}m_1^5 - \frac{\frac{255}{128}m_1^3 + \frac{7543}{1536}m_1^4 - \frac{134525}{8192}m_1^5}{1 - 4m_1 - \frac{37}{8}m_1^2 - \dots} \right] \times \frac{a}{a'}\sin 3\mathrm{D}$$

$$+1''{\cdot}6172 \quad +0{\cdot}1199 \quad -0''{\cdot}0006 \quad +0''{\cdot}0005$$

$$-0''{\cdot}8624 \quad -0''{\cdot}1718 \quad +0''{\cdot}0470$$

$$\left[+\frac{\frac{75}{128}m_1^4 + \frac{2797}{2048}m_1^5 - \frac{18415}{8192}m_1^5 \times \frac{1}{1 - 4m - \dots}}{+0''{\cdot}0093} \right] \times \frac{a}{a'}\sin 5\mathrm{D}.$$

For the sake of comparison I give Delaunay's expressions* for the same terms; they are expanded in powers of m.

$$-\left[\frac{15}{8}m+\frac{93}{8}m^2+\frac{6887}{128}m^3+\frac{137197}{512}m^4+\frac{4628333}{3072}m^5+\frac{63106813}{8192}m^6\right.$$

$$-74''{\cdot}0235 \quad -34''{\cdot}3297 \quad -11''{\cdot}8852 \quad -4''{\cdot}4276 \quad -1''{\cdot}8621 \quad -0''{\cdot}7122$$

$$\left.+\frac{10835537159}{196608}m^7\right]\frac{a}{a'}\sin D$$

$$-0''{\cdot}3811$$

$$+\left[\frac{15}{32}m^2-\frac{5}{8}m^3-\frac{259}{16}m^4-\frac{666249}{6144}m^5\right]\frac{a}{a'}\sin 3D$$

$$+1''{\cdot}3843 \quad -0''{\cdot}1381 \quad -0''{\cdot}2675 \quad -0''{\cdot}1340$$

$$+\left[\frac{75}{128}m^4+\frac{3911}{1024}m^5\right]\frac{a}{a'}\sin 5D$$

$$+0''{\cdot}0097 \quad +0''{\cdot}0047$$

The gain in accuracy by the former mode of expression is evident; the terms there are carried only as far as $m_1{}^5$, while Delaunay's go as far as m^7.

The equations for obtaining a_1 a_{-1} involve largely a_2 a_{-2}, which go to form the coefficient of the Variation. It has been mentioned that the expansions of these quantities are much more convergent when expressed in terms of μ, § (3). Hence it is probable that the resulting expressions for the coefficients of the Parallactic Inequality would be also rendered more convergent when expressed in terms of this quantity. This inequality thus transformed becomes,

$$-\left[\frac{\frac{15}{8}\mu+\mu^2-\frac{2158}{128}\mu^3-\frac{175705}{9216}\mu^4+\frac{3261161}{442368}\mu^5}{1-\frac{13}{3}\mu-\frac{133}{72}\mu^2+\frac{91}{216}\mu^3-\frac{701891}{82944}\mu^4}+\frac{35}{1024}\mu^4+\frac{785}{12288}\mu^5\right]\frac{a}{a'}\sin D.$$

$$-131''{\cdot}1311 \quad -5''{\cdot}8109 \quad +8''{\cdot}1438 \quad +0''{\cdot}7648 \quad -0''{\cdot}0246 \quad -0''{\cdot}0009 \quad -0''{\cdot}0001$$

[In making the transformation, both numerator and denominator of the expression given at the beginning of this section must be multiplied by m_1, and, in the result, for m_1 must be put $\mu/(1+\frac{1}{3}\mu)$. It is in that form that the expression occurs.] The series in both numerator and denominator seem slightly more convergent, and the numerical result is a little closer to the final value.

12. To exhibit the values of the coefficients obtained by using these several expressions, a table is given. In column (i.) is the argument of the term; in (ii.) the coefficients found by using the numerical value of m_1 from the start; (iii.) those found by

* Delaunay, *Mém. cit.* pp. 847, 856, 860.

putting the numerical values of α_1 β_1, etc. in the expressions of § (10); (iv.) gives the values from the expressions in terms of m_1 in § (ii.); (v.) those in terms of μ in the same section; in (vi.) are Delaunay's values. No estimate of neglected terms is taken into account.

i.	ii.	iii.	iv.	v.	vi.
D	−128″·069	−128″·073	−128″·056	−128″·059	−127′·621
3D	+ 0·750	...	+ 0·749	...	+ 0·845
5D	+ 0·008	...	+ 0·009	...	+ 0·014

The column (ii.) gives the final values to be used; (iii.), (iv.), or (v.) may serve also as checks on the numerical work. It may be noticed that in obtaining the numerical values from the outset it is only necessary to substitute for m_1 its value. The ratio a/a' occurs only in its first power; it may therefore be kept as a factor right through, and its numerical value substituted as the final step.

13. The expansions I have given for the parallactic terms, when transformed, agree with those of Delaunay as far as the terms of the sixth order. The numerical multipliers of $m^5.a/a'$ differ by small quantities; these differences would not, however, cause differences in the coefficients expressed in seconds of more than 0″·004. As the coefficient of the term with argument 5D is very easily obtainable by the methods indicated, I am inclined to attribute some small numerical errors to Delaunay. Newcomb suggests * that Delaunay's multiplier of $m^7 a/a'$ may be wrong. I have not yet been able to test Delaunay's multipliers of $m^6.a/a'$ and $m^7.a/a'$ owing to the large amount of labour they entail. But that the numerical values given in column (ii.) of the table do not differ by more than one or two thousandths of a second from the true values can be seen during the progress of the work necessary for finding those values.

14. The corresponding terms in the expression for the Moon's parallax, which I have obtained, are :—

$$-\frac{1}{a}\left[\frac{\frac{15}{16}m_1+\frac{3}{8}m_1^2-\frac{1869}{256}m_1^3}{1-4m-\frac{37}{8}m^2-\ldots}\right]\frac{a}{a'}\cos \mathrm{D}.$$

$$-1''\cdot0291 \quad -0''\cdot0333 \quad +0''\cdot0524 \quad = -1''\cdot0100.$$

$$+\frac{1}{a}\left[\frac{25}{64}m_1^2+\frac{65}{256}m_1^3-\frac{459}{256}\cdot\frac{m_1^4}{1-4m_1-\ldots}\right]\frac{a}{a'}\cos 3\mathrm{D}.$$

$$+0''\cdot0223 \quad +0''\cdot0012 \quad -0''\cdot0139 \quad = +0''\cdot0096.$$

* Newcomb, *Transformation of Hansen's Lunar Theory*, Astronomical Papers for Use of American Ephemeris, vol. i., part ii.

Delaunay's expressions * for the same being

$$-\frac{1}{a}\left[\frac{15}{16}m+\frac{81}{16}m^2+\frac{5817}{256}m^3\right]\frac{a}{a'}\cos \mathrm{D}.$$

$$-0''{\cdot}6136 \quad -0''{\cdot}2479 \quad -0''{\cdot}0832 \quad = -0''{\cdot}9447.$$

$$+\frac{1}{a}\left[\frac{25}{64}m^2-\frac{115}{128}m^3\right]\frac{a}{a'}\cos 3\mathrm{D}.$$

$$+0''{\cdot}0191 \quad -0''{\cdot}0033 \quad = +0''{\cdot}0158.$$

The coefficients obtained by using numerical values from the start are $-1''{\cdot}0010$ and $+0''{\cdot}0080$, to which the former pair approximate closely.

15. A comparison of these new values with those given by Hansen cannot be complete owing to the fact that Hansen's numerical values contain the terms due to eccentricity and inclination. But a rough estimate of the uncalculated parts of these series may be made from Delaunay's expressions, since they are carried to a higher degree of approximation for numerical purposes than the terms we have been considering. The whole coefficient of the Parallactic Inequality in the longitude then becomes

$$127''{\cdot}30$$

Newcomb's method of transformation † shows that this coefficient when expressed in Hansen's notation is

	$124''{\cdot}54$
Hansen's coefficient is	$124{\cdot}43$
Diff.	$0{\cdot}11$

The difference is probably partly due to a wrong estimate of the omitted terms in Delaunay's coefficient.

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Secular Perturbations of the Earth's Orbit by Mars.

By R. T. A. Innes.

In Professor Hill's "Theory of *Jupiter* and *Saturn*" he mentions that the terms arising from the fifth powers of the eccentricities and inclination amount to one per cent. of those of the first order in the case of *Mars* and the Earth, and that a computation by Gauss's method would be desirable. I have therefore

* Delaunay, *Mém. cit.* pp. 922, 923.
† Newcomb, *Mém. cit.*

made it. To let the comparison with Le Verrier's results be an exact one, I have used the same elements as Le Verrier in "Les Annales del'Obs. de Paris," *Mémoires*, tome ii. The formulæ used are Professor Hill's, in the appendix to his paper on Gauss's method, published in the Astronomical Papers of the American Ephemeris, where the signification of the symbols will be found.

As a guide to future computers, I give the chief figures involved. This will also lead more easily to the detection of any errors I may have made.

Le Verrier's elements and Professor Hill's formulæ give:

$$\Pi = 51^\circ\ 58'\ 55''\cdot25,\ \sin\Pi = [9\cdot8964256]$$

$$\Pi' = 284\ 55\ 5\cdot75,\ \tan\Pi' = [n0\cdot5744319]$$

$$K = 127\ 3\ 22\cdot70$$

$$K' = K + 53''\cdot59$$

$$k = [9\cdot9997883]$$

$$k' = [9\cdot9999849]$$

$$r = 1 - [8\cdot2245450]\cos E$$

$$\tan\frac{v}{2} = [0\cdot0072840]\tan\frac{E}{2}$$

$$\nu' = \frac{\sqrt{3}}{32m_1^2}\left\{1 + 2\frac{\lambda^4}{m_1^2m_{11}^2} + 4\frac{}{m_1^6m_{11}^4m_{111}^2} + \right\}$$

$$\mathrm{Log}\frac{\sqrt{3}}{32} = 8\cdot7334106\cdot49.$$

$$A = r^2 + [9\cdot4534182]r\cos(v + 127^\circ\ 3'\ 22''\cdot70) + 2\cdot3216353$$

$$B\cos\epsilon = 0\cdot2165194\cdot5 + [0\cdot1826853]r\cos(v + 127^\circ\ 3'\ 22''\cdot70)$$

$$B\sin\epsilon = [0\cdot1809850]r\sin(v + 127^\circ\ 4'\ 16''\cdot29)$$

$$C = 0\cdot0201929\cdot5 = [8\cdot3051998]$$

$$F_1 = [9\cdot6379998]\frac{B^2 - AC}{3q}$$

$$F_2 = [n8\cdot9713730]\frac{B\sin\epsilon}{q}$$

$$F_3 = F_2 \times [8\cdot5095314]r\cos(v + 51^\circ\ 58'\ 55''\cdot25)$$

$$J_1 = 1 - [7\cdot0186092]\sin^2(v + 51^\circ\ 58'\ 55''\cdot25) - p[9\cdot9390298]$$

$$J_2 = [8\cdot7903880]r\sin(v + 127^\circ\ 3'\ 22''\cdot70) - [7\cdot0186092]\sin(v + \Pi)\cos(v + \Pi)$$

$$J_3 = [8\cdot5090779]r\sin(v + 51^\circ\ 58'\ 55''\cdot25) - [7\cdot1963299]r^2$$

To find $A_0^{(c)}$ $B_0^{(c)}$ $C_0^{(c)}$, etc., etc., I divided E into sixteen parts, and the differences between the sums S and S_1 would seem to show that even division into sixteen parts is not too much.

The values of the functions involved are given in the following tables:—

E	r	v	A	B cos ε	Log B sin ε
°		° ′ ″			
0	0·98322954	0 0 0·00	3·1200687	−0·6858181	0·0755814
22½	0·98450612	22 52 14·19	3·0488700	−1·0810004	9·8739366
45	0·98814149	45 41 0·71	3·0196127	−1·2763040	9·2765785
67½	0·99358222	68 23 26·31	3·0367939	−1·2419961	*n* 9·6040431
90	1·00000000	90 57 39·32	3·0978414	−0·9832986	*n* 9·9706386
112½	1·00641778	113 23 5·76	3·1934788	−0·5395969	*n* 0·1232717
135	1·01185851	135 40 31·73	3·3091284	+0·0215597	*n* 0·1826155
157½	1·01549388	157 51 53·67	3·4271406	+0·6147402	*n* 0·1727356
180	1·01677046	180 0 0·00	3·5295059	+1·1496383	*n* 0·0901494
202½	1·01549388	202 8 6·33	3·6006222	+1·5448206	*n* 9·8968887
225	1·01185851	224 19 28·27	3·6296803	+1·7401243	*n* 9·3610636
247½	1·00641778	246 36 54·24	3·6123004	+1·7058160	9·5577875
270	1·00000000	269 2 20·68	3·5511707	+1·4471189	9·9513518
292½	0·99358222	291 36 33·69	3·4556156	+1·0034174	0·1097902
315	0·98814149	314 18 59·29	3·3401649	+0·4422605	0·1708793
337½	0·98450612	337 7 45·81	3·2223515	−0·1509200	0·1607256
S	8·00000000	1260 0 0·00	26·5971730	+1·8552810	
S	8·00000000	1440 0 0·00	26·5971730	+1·8552808	

E	p	q^2	r	Π	log m^2	log n^2	log μ
0°	1·0332919	0·4598031	−0·1467474	−28° 4′ 39″·09	9·9941810	9·8022124	9·9504225
22½	1·0095590	0·4636798	−0·1820422	−35 12 32·37	9·9908244	9·8232810	9·9545353
45	0·9998066	0·4650408	−0·1973554	−38 29 8·24	9·9890222	9·8323854	9·9562339
67½	1·0055336	0·4635310	−0·1891672	−36 49 40·95	9·9899544	9·8278233	9·9553892
90	1·0258828	0·4598188	−0·1589229	−30 38 35·56	9·9930618	9·8099969	9·9519690
112½	1·0577619	0·4552393	−0·1127466	−21 32 5·19	9·9965827	9·7812503	9·9461238
135	1·0963118	0·4511423	−0·0596507	−11 21 11·10	9·9990520	9·7451285	9·9383598
157½	1·1356492	0·4482996	−0·0089750	− 1 42 48·42	9·9999784	9·7063832	9·9296824
180	1·1697710	0·4466968	+0·0318289	6 7 11·99	9·9997247	9·6710386	9·9215633
202½	1·1934764	0·4458017	+0·0581844	11 16 21·16	9·9990654	9·6455903	9·9156400
225	1·2031624	0·4451073	+0·0680766	13 15 9·53	9·9987079	9·6352922	9·9132303
247½	1·1973691	0·4445748	+0·0611672	11 54 30·39	9·9989569	9·6423159	9·9148744
270	1·1769926	0·4447342	+0·0381479	7 23 24·16	9·9995985	9·6649389	9·9201483
292½	1·1451409	0·4463320	+0·0009097	0 10 29·25	9·9999998	9·6982042	9·9278176
315	1·1066573	0·4497624	−0·0467645	− 8 55 8·66	9·9994151	9·7357938	9·9362916
337½	1·0673862	0·4546475	−0·0987180	−18 47 6·67	9·9974013	9·7719362	9·9441601
S	8·7918764	3·6221057	−0·4713877	−90 42 56·97	9·9727632	7·8967867	9·4882187
S′	8·7918763	3·6221057	−0·4713880	−90 42 52·80	9·9727633	7·8967844	9·4882228

Π

E	log λ^3	log ψ'	log $\chi(r)$	$\psi(r)$	log V	$F_1\chi(r)$	$F_2\chi(r)$
°							
0	8·3430497	8·8305826	8·7453586	0·0741897	9·2013155	0·02167189	−0·00914170
22½	8·2919657	8·8228184	8·7132695	0·0696371	9·1650909	0·01831754	−0·00531459
45	8·2660437	8·8196212	8·7000483	0·0678487	9·1502562	0·01707298	−0·00130093
67½	8·2793701	8·8212164	8·7066194	0·0687315	9·1580297	0·01778349	+0·00281187
90	8·3254405	8·8276703	8·7332790	0·0724400	9·1883721	0·02054752	+0·00698228
112½	8·3845610	8·8386693	8·7790226	0·0793065	9·2394459	0·02569346	+0·01108019
135	8·4410573	8·8532282	8·8401553	0·0895794	9·3065140	0·03361226	+0·01468882
157½	8·4872642	8·8694241	8·9090100	0·1028612	9·3809596	0·04428206	+0·01687852
180	8·5205743	8·8845011	8·9739748	0·1173140	9·4504900	0·05643419	+0·01623644
202½	8·5407042	8·8954511	9·0217167	0·1292960	9·5013472	0·06694892	+0·01162539
225	8·5480973	8·8998932	9·0412263	0·1345584	9·5222568	0·07179179	+0·00354348
247½	8·5430985	8·8968888	9·0279064	0·1309423	9·5084480	0·06873565	−0·00540872
270	8·5256601	8·8871396	8·9853503	0·1200587	9·4634389	0·05925082	−0·01213458
292½	8·4956140	8·8729053	8·9238828	0·1059951	9·3976724	0·04721478	−0·01514318
315	8·4533278	8·8570930	8·8564791	0·0925550	9·3245542	0·03607853	−0·01486756
337½	8·4007099	8·8423608	8·7944432	0·0817742	9·2562021	0·02748428	−0·01252289
S.	7·4232509	0·7597292	8717	439		0·31646000	+0·00400625
S′.	7·4232877	0·7597342	8706	439		0·31646018	+0·00400660

E	$F_3\chi(\tau)$	J_1	J_2	J_3	$\log a a_1^3 \cos\phi' r\varrho^{-\frac{3}{2}}$	$\frac{a}{r}R_0$.	$\frac{a}{r}S_0$.	$\frac{a}{r}W_0$.
°								
0	−0·000178949	+0·1014009	+0·0479185	+0·02349328	0·7764402	0·2258587	−0·0091085	+0·0212508
22½	−0·000044194	+0·1217004	+0·0301830	+0·02916268	0·7724458	0·2138680	−0·0053317	+0·0249943
45	+0·000005543	+0·1301228	+0·0078447	+0·03008817	0·7724555	0·2100120	−0·0011380	+0·0252158
67½	−0·000045662	+0·1253943	−0·0158766	+0·02612875	0·7766056	0·2141949	+0·0031531	+0·0222048
90	−0·000180115	+0·1081083	−0·0375080	+0·01788717	0·7837667	0·2262802	+0·0072616	+0·0156809
112½	−0·000348768	+0·0807171	−0·0537719	+0·006609190	0·7919788	0·2459222	+0·0108250	+0·0049448
135	−0·000476155	+0·0472645	−0·0620825	−0·005962846	0·7992281	0·2719968	+0·0133181	−0·0106056
157½	−0·000480553	+0·0128394	−0·0610082	−0·01794022	0·8042171	0·3017932	+0·0140885	−0·0305409
180	−0·000328671	−0·0172021	−0·0505834	−0·02749054	0·8067070	0·3305172	+0·0125849	−0·0518088
202½	−0·000104440	−0·0381197	−0·0323730	−0·03315980	0·8072503	0·3519508	+0·0087020	−0·0681554
225	+0·000012731	−0·0466033	−0·0092457	−0·03408497	0·8065391	0·3604873	+0·0029870	−0·0725891
247½	−0·000084221	−0·0413425	+0·0151187	−0·03012524	0·8048474	0·3535068	−0·0034062	−0·0625137
270	−0·000304922	−0·0232433	+0·0368684	−0·02188354	0·8018745	0·3326480	−0·0089798	−0·0422440
292½	−0·000466545	+0·0047662	+0·0526579	−0·010614566	0·7971314	0·3034080	−0·0124533	−0·0195472
315	−0·000472020	+0·0382799	+0·0601789	+0·00196604	0·7905904	0·2726627	−0·0133480	−0·0003515
337½	−0·000348180	+0·0721729	+0·0584620	+0·01394311	0·7831251	0·2458188	−0·0119997	+0·0131515
	−0·001922558		−0·0066081	−0·01598724	6·3376016	2·2304629	−0·0035771	−0·1154515
	−0·001922563		−0·0066081	−0·01599610	6·3376016	2·2304627	0·0035776	−0·1154618

From which

$A_0^{(c)} = +0{\cdot}2788078$	$\text{Log} = 9{\cdot}4453050$
$\frac{1}{2}A_1^{(c)} = -0{\cdot}0263243$	$[n\ 8{\cdot}4203568$
$\frac{1}{2}A_1^{(s)} = -0{\cdot}0266550$	$n\ 8{\cdot}4257783$
$\frac{1}{2}A_2^{(c)} = -0{\cdot}0003187$	$n\ 6{\cdot}5033400$
$B_0^{(c)} = +0{\cdot}0004472$	$6{\cdot}6504737$
$\frac{1}{2}B_1^{(c)} = -0{\cdot}0054333$	$n\ 7{\cdot}7350613$
$\frac{1}{2}B_1^{(s)} = +0{\cdot}0040225$	$7{\cdot}6044961$
$\frac{1}{2}B_2^{(c)} = +0{\cdot}0006491$	$6{\cdot}8122989$
$\frac{1}{2}B_2^{(s)} = +0{\cdot}0002348$	$6{\cdot}3707906$
$C_0^{(c)} = -0{\cdot}0144321$	$n\ 8{\cdot}15933$
$\frac{1}{2}C_1^{(c)} = +0{\cdot}0186839$	$8{\cdot}271467$
$\frac{1}{2}C_1^{(s)} = +0{\cdot}0149785$	$8{\cdot}17547$

And we have

$$\frac{m'n}{1+m} = [9{\cdot}6844068].$$

These give

		Le Verrier's Values.
	"	"
$\left[\frac{de}{dt}\right]_{00}$	$-0{\cdot}018146$	$-0{\cdot}01816$
$e\left[\frac{d\varpi}{dt}\right]_{00}$	$+0{\cdot}018876$	$+0{\cdot}01888$
$\left[\frac{dp}{dt}\right]_{00}$	$+0{\cdot}007700$	$+0{\cdot}00733$
$\left[\frac{dq}{dt}\right]_{00}$	$-0{\cdot}008770$	$-0{\cdot}00832$
$\left[\frac{dL}{dt}\right]$	$-0{\cdot}270876$	$-0{\cdot}2697$

The differences are exceedingly small and tend to prove the general correctness of the calculation.

The differences in

$$\left[\frac{dp}{dt}\right]_{00} \text{ and } \left[\frac{dq}{dt}\right]_{00}$$

probably arise from the non-convergency of the function

$$\frac{m'}{2\mu}C$$

in Le Verrier's expansion. The function

$$\frac{m'}{2\mu}G$$

gives for the

1st order	$-0''\cdot0096363$
3rd „	$-0\cdot0012981$
5th „	$-0\cdot0001487$

from which we may estimate that the terms of higher orders will not exceed $-0''\cdot00002$.

The equation

$$\left[\frac{da}{dt}\right]_{00} = 0$$

gives

$$\sin\phi\tfrac{1}{2}A_1^{(s)} + \cos\phi B_0^{(c)} = 0,$$

Substituting the respective numbers I get

$$-0\cdot000447016 + 0\cdot000447108 = 0,$$

error—

$$+0\cdot000000092.$$

In making a similar operation on the figures, Professor Hill has given in his example of *Mercury*, perturbated by *Venus*, the difference is more considerable—viz.,

$$-0\cdot00075,$$

probably through a misprint of a figure in the value of R_0 or S_0.

In some parts of the calculations I have used eight decimals, and S and S_1 refer to the original numbers.

Sydney: 1891 *November* 1.

On the Spectrographic Method of determining the Velocity of Stars in the Line of Sight. By H. C. Vogel, Foreign Associate.

The experiments made at Potsdam in 1887 showed that, as a result of the extremely sensitive photographic methods employed, a sufficiently great dispersion could be made use of to readily detect and measure the displacement of the spectral lines produced by the motion of the stars in the line of sight. It very soon became clear that the measurement of the stellar spectra admitted of a far greater exactness than the direct observations, and that the disturbances of the atmosphere—the chief cause of the difficulties of the direct method—exert their influence in a lesser degree on the photograph. The very numerous measurements on more than two hundred negatives of forty-seven stars,

which are now available, have confirmed this result, and show further that the exactness of the measurements far surpasses the expectations based on the first plates taken with a provisional apparatus, and that the definitive observations have reached a degree of accuracy which in some cases is surprising.

This great accuracy has been secured by an advantageous construction of the apparatus, by its very exact adjustment, and especially by the peculiar methods adopted in measuring the photographs. I have already published several communications on this spectrographic method, viz. one in *Ast. Nach.*, No. 2896, a further one announcing the discovery of the motion of *α Virginis* (*Ast. Nach.*, No. 2995), and more recently an article on the employment of iron as a comparison spectrum in spectrographic researches (*Sitzungsberichte der Akademie zu Berlin*, June 4, 1891).

The reduction of the observations and measurements are at present nearly completed, but the passing through the press will still require several months, and therefore I now take the liberty of presenting a short review of the investigation and its chief results, in the hope that this summary, and more especially the explanation of the method of measuring the plates, may be of value and interest to many who are employed in similar lines of work.

In the construction of the apparatus the following points were taken into consideration: great stability for the smallest possible weight; suitable dimensions of prisms, collimator, and camera objectives, in order to preserve sufficient brightness with the greatest possible dispersion; accurate adjustment of the photographic plate in the focal plane of the camera objective; and exact keeping of the star on the slit of the spectrograph. I have endeavoured to satisfy the first two conditions by having the frame made of cast steel, and by giving it a form which offered the greatest possible resistance to flexure. The most suitable dimensions for collimator and camera objective, for the 12-inch refractor to which the spectrograph was to be applied, were found to be 408 mm. focal length for 34 mm. aperture.

The two Rutherfurd compound prisms have the following dimensions: Height (length of the refracting edge) 35 mm., breadth (perpendicular distance from the refracting edge of the flint-glass prism) 45 mm. They are of the most colourless glass obtainable, and the dispersion from F to H amounts for each to about 5°. The camera is constructed of sheet steel; the plate-holders are of brass and can be rigidly connected with the camera. The adjustment of the photographic film in the focal plane of the camera objective is accomplished by a motion of the latter, and can be effected with an accuracy of a fraction of a millimetre. In order to facilitate keeping the image of the star exactly on the slit, a small telescope is so connected with the apparatus that it receives the light which is reflected from the front side of the first prism.

The slit, illuminated by the Geissler tube, which furnishes the comparison spectrum, appears in this telescope as a narrow line of light with the star in the middle, and may be readily so held by means of the slow motions of the refractor. A cylindrical lens is not employed, since, with the slit set parallel to the line of the diurnal motion, a slight widening of the linear stellar spectrum can be readily effected by changing the rate of the driving clock. A very important point in the adjustment of this apparatus is that the optical axis of the collimator shall fall in the prolongation of that of the refractor; this is easily and accurately effected by three screws.

By means of a table giving the proper setting for various degrees of temperature the plane of the slit is, before each observation, adjusted to lie in the focus for the rays of the wave-length of $H\gamma$, for which the prisms are set at the angle of minimum deviation, and which appear on the middle of the negative. It is of great importance that the latter be precisely adjusted in the focus of the camera objective. This is likewise effected before each observation according to a table with the argument of the thermometer attached to the instrument. This table has been determined by many experiments, as well as by artificial warming of the apparatus. A false adjustment, which exerts a great influence on the sharpness of the images, and hence on the subsequent measurements, can be at once detected by the lack of distinctness of the image of the artificial hydrogen line, and, similarly, changes of the apparatus which occur during the exposure reveal themselves in the altered appearance of the comparison line.

In nearly all the observations hydrogen has been chosen for the comparison spectrum. The Geissler tube was placed directly in the cone of rays of the refractor, at a distance of 40 cm. from the slit, and was set at right angles to the optical axis of the refractor as well as to the slit, and therefore its light is to be regarded as dispersed on reaching the slit. The tubes used were very thin, so that only a comparatively small amount of light (17 per cent.) was lost in passing through them.

A further advantage of this arrangement of the Geissler tube lies in the fact that an exact adjustment of its position is not necessary, and any slight changes in its place during the exposure, due to an altered position of the instrument, can exert no injurious influence; for one can readily see that with an adjustment even several degrees false, the slit would still appear fully illuminated.

At first the time of exposure was regulated according to the brightness of the star and to the chosen width of slit, but later it appeared advantageous to leave the width unchanged at 0·02 mm. and to give a uniform exposure of one hour. The observer then, however, varied the width of the spectrum to correspond with the brightness of the star. The performance of the apparatus has been tested on the Sun by direct observa-

tions and by photographs. In the vicinity of Hγ, where the spectrum is sharpest, nearly all the lines are visible which are contained in Rowland's large photograph of the solar spectrum, in spite of the comparatively small dimensions of the apparatus. Of course very close double lines cannot be separated if the width of the slit be greater than that corresponding to the distance between the lines.

One perceives less detail on the photograph than by direct observation, since, on account of the extraordinary delicacy of the lines, some details are lost through the coarseness of the silver grains. The excellent prisms would admit of a much greater magnification of the spectrum by the use of a camera objective of longer focus, and thus make possible the production of still sharper photographs, if the light-power of the refractor were not too small and the observations therefore necessarily limited to the brightest stars. I have given in *Ast. Nach.*, No. 2896, a collection of about fifty lines on one of the solar negatives which lie between 431·4 and 436·8 $\mu\mu$, and may remark further that my assistant, Dr. Scheiner, has been able to measure 290 lines in the spectrum of *α Aurigæ* between 412·4 and 466·8 $\mu\mu$.

In order to decide further as to the correct adjustment of the apparatus, numerous photographs of the Moon's spectrum, with that of hydrogen for comparison, have been made, and show an absolute coincidence of the corresponding lines. Two photographs of *Venus* are not without interest as showing the accuracy of the measures as well as the correctness of the adjustment:

	Observed Velocity.	Calculated Velocity.	
1889 Jan. 2	−7·8	−7·4	English miles.
Feb. 10	−7·4	−7·8	

The measurement of the spectra is accomplished with the aid of a microscope under the employment of magnifying powers from 7 to 35.

The table of the microscope carries a sliding apparatus, movable by a fine micrometer screw of 0·25 mm. pitch, to which the negative is firmly clamped. The periodic and progressive errors of this screw have been accurately determined. By means of a large number of measurements in the solar spectrum it has been found that one revolution of the screw corresponds to a difference in wave-length of 0·324 $\mu\mu$. With this value can now be computed g, the value in miles per second of velocity of one revolution of the screw, from the formula $g = \frac{\Delta\lambda}{\lambda} V$, where V is the velocity of light and λ the wave-length of Hγ, 434·07 $\mu\mu$. The result is $g = 139{\cdot}13$ miles. This value, strictly speaking, holds good only for the temperature at which this standard photograph

of the solar spectrum was taken, since the dispersion varies with the temperature. A small correction has, therefore, been applied to all the photographs made at a different temperature. For stars of the second type, however, a method of measurement has been chosen which takes for a basis this standard solar negative, as I shall presently show.

In the case of the spectra of Classes II. and III., containing numerous lines, it suggested itself, in determining the displacement between the artificial Hγ and the star line, to connect the former by differential measurements with the other neighbouring lines, not only in order to obtain an increased degree of accuracy, but also indeed from necessity, since, in the generally very small displacements, it wholly or partly concealed the star line. All the difficulties which at first arose in the way of direct measurement of the plates, into the details of which I cannot here enter, were thereby overcome in that I simultaneously with the star spectrum also measured a standard solar spectrum taken with the spectrograph. The plate with the solar spectrum is cut off lengthwise, and so laid upon the star spectrum that the two appear in the microscope one above the other, and separated only by a small space.

The solar negative is observed through the glass, since, in order to avoid parallax, it must be inverted so that the two gelatine films are in contact. It is therefore cut off in order to obviate the effect of the unavoidable impurities in its gelatine film, through which otherwise the star spectrum would have to be observed. It can be then readily brought about that the lines of the one spectrum form the prolongation of those of the other.

The setting upon one of the threads of the system in the eyepiece of the microscope is effected solely by the motion of the above-mentioned sliding apparatus, to which both negatives are firmly attached. In the measurements four settings were usually made on a Sun line, then an equal number on the corresponding star line, those lines being sought out which, generally three on each side, lie nearest Hγ. The Hγ line in the star has only been measured when it was fully separated from the artificial line; but generally, instead of upon it, settings were made six or eight times on the solar Hγ, and an equal number on the artificial Hγ on the star plate.

The difference of the readings on the two spectra gives, in the mean, their displacement as compared with each other as they were brought together under the microscope; this mean, being applied to the difference of the settings on Hγ in the solar negative and the artificial Hγ due to the Geissler tube, gives the actual displacement of the star lines referred to the artificial line.

In most cases the star spectra have been photographed at other temperatures than the Sun, and the differences Sun–star are unequal and show a rate of progression. Under the assump-

tion of a simple proportional change, which is fully admissible for the small distances which alone come into question, a reduction to the dispersion of the solar negative must first be made, which has been done by the method of least squares.

The great advantage of this method of measurement lies in the fact that the unavoidable distortions in the sensitive film, which, owing to the smallness of the quantities to be measured, might easily come into consideration, are as much as possible eliminated, and also, further, that every prejudice of the observer is entirely eliminated, since the amount of the displacement is not directly obtained, but only after computation.

Most stars of the first spectral class show, in addition to the broad and strongly-marked hydrogen lines, a great number of other lines, so fine, however, that they can be distinctly recognised only in case of the brighter stars. The majority of these lines belong to the iron spectrum, and can be readily identified with solar lines; the measurement of the displacement can therefore either be made by reference to the solar negative, or, under the precautions which I have already stated in the article quoted, the iron spectrum may be used for comparison.

In the case of fainter stars of this class, however, one is restricted to the hydrogen lines, and if the more or less distinctly pronounced maximum of intensity lies outside of the artificial line, then the measurement presents no difficulty; but this is seldom the case, since the breadth of the maximum intensity corresponds to a difference in wave-length of about 0·03 $\mu\mu$, and a displacement of the artificial line only enough to bring its edge into coincidence with the position of maximum intensity would presuppose a velocity of fourteen miles. This maximum of intensity is generally not very sharply bounded, and, with this gradual decrease of the blackening on the negative, it could be accurately observed only in cases of a much greater displacement. The difficulties of this case were therefore insurmountable until a special procedure was thought out, which consisted in covering the Hγ star line, along with the artificial line, by a comparatively broad strip which was brought exactly over the middle of the Hγ line in the star. If, then, the thread of the micrometer be set on the middle of this strip, and the latter be then removed and the thread set on the artificial line, the difference in the readings will give the displacement.

The accompanying figure will make the procedure clearer. M M is the intensity curve of the star spectrum in the vicinity

of Hγ, K is the artificial Hγ, displaced from the middle by the star's motion, and S is the strip covering the middle of the position of maximum intensity and the line K.

With a magnification such that the silver grains are readily perceptible, the setting of the strip over the middle of the broad Hγ line can, after some practice, be accurately accomplished by fastening the attention upon the density of the silver precipitate on the plate to right and left of the strip. From a series of small glass plates, upon each of which had been photographically produced a dark strip of differing width (from 0·05 to 0·20 mm.), the suitable one was chosen and laid directly upon the star negative, and fastened by a simple mechanical contrivance which allowed of its being moved backwards and forwards by a fine screw.

When the line in the star spectrum has been very broad and without a distinct position of maximum intensity, I have employed still another method in the measurement of stars with large velocities. A number of lines of varying breadth and blackness, but narrower than the strips just mentioned, were photographed upon small glass plates, and that one was then selected which most nearly equalled in breadth and blackness the artificial Hγ on the negative to be measured. The plate was then laid upon the star negative, and by the fine screw set upon the broad star line, so as to be symmetrical with the artificial Hγ. In the figure the star line is represented by the dipping of the curve M M, K is the line from the Geissler tube, to which is symmetrically set the line S on the glass plate. The micrometric measurement of the distance K S gives the double value

of the displacement. By practice much can here be gained in accuracy; attention to the silver grains is in this case also important. It has shown itself to be of advantage to repeat the measurements on different days, since very soon a certain habit of perception sets in which gives the measurements on a single day the appearance of an accuracy which is somewhat illusory.

The first result of any importance which the spectrographic method furnished was the proof of the influence of the Earth's motion on the displacement, which the earlier direct observations had failed to show with certainty. I append here a few examples:—

α Aurigæ.

Date.	Obs. Vel.	Earth's Vel.	Vel. of Star red. to ⊙.
1888 Oct. 6	− 3·5	−15·4	+11·9
22	+ 2·9	−13·0	+15·9
24	+ 3·8	−12·6	+16·4
25	+ 3·5	−12·4	+15·9
28	+ 3·8	−11·8	+15·6
Nov. 9	+ 9·2	− 8·9	+18·1
Dec. 1	+10·8	− 2·9	+13·7
13	+15·6	+ 0·7	+14·9
1889 Jan. 2	+20·2	+ 6·6	+13·6
Feb. 5	+30·8	+14·3	+16·5
May 6	+33·2	+17·0	+16·2
Sept. 15	− 3·6	−16·8	+13·2

α Tauri.

1888 Oct. 28	+18·1	− 9·5	+27·6
Nov. 10	+24·9	− 5·9	+30·8
Dec. 4	+30·6	+ 1·8	+28·8
1890 Jan. 9	+43·7	+12·3	+31·4

α Ophiuchi.

1888 Sept. 30	+27·6	+14·1	+13·5
1889 June 7	+ 9·7	− 1·0	+10·7

α Ursæ Majoris.

1888 Nov. 7	−17·0	−11·9	− 5·1
9	−18·1	−11·9	− 6·2
1889 May 4	+ 5·2	+11·8	− 6·6
22	+ 3·4	+11·2	− 7·8

A further result of the new method was the discovery of the changes in the motion of *Algol*, and thereby the proof of the existence of a dark satellite, for the determination of which the most delicate measurements were necessary. The discovery of the periodic motion of *α Virginis* then followed.

As an example of the accuracy of the method of symmetrical setting, which has been applied for stars with broad and ill-defined lines, and generally in the case of *α Virginis*, I give here the results of the measurements, with the period which I have deduced from them :—

α Virginis. Velocity in the line of sight − velocity of system

$$= 56{\cdot}7 \sin\left(\frac{t-t_0}{p}\right) 360°, \text{ in miles.}$$

Velocity of system = − 9·2 miles. Period, p, = 4·0134 days.

Epoch, t_0, = 1890 May 4, $10^h{\cdot}50$ Potsdam, m.t.

Date.	Obs. Vel. less Vel. of Translation.	Calc. Vel.	O.−C.
1889 April 21	−48·4	−53·5	+5·1
29	−54·3	−52·1	−2·2
May 1	+50·7	+52·6	−1·9
1890 April 4	− 6·5	−12·4	+5·9
9	−55·3	−56·2	+0·9
10	+ 6·0	+10·6	−4·6
11	+50·7	+56·2	−5·5
13	−57·6	−56·2	−1·4
15	+62·7	+56·2	+6·5
May 1	+53·5	+56·7	−3·2
4	− 3·7	− 1·8	−1·9
7	−61·3	−56·7	−4·6
8	− 0·5	− 1·4	+0·9
9	+63·2	+56·7	+6·5
17	+61·8	+56·7	+5·1
18	+ 6·0	+ 5·5	+0·5
23	−65·5	−56·2	−9·3
24	− 1·4	− 5·5	+4·1
25	+53·0	+56·2	−3·2
26	+ 5·1	+ 5·5	−0·4
27	−52·6	−56·2	+3·6
28	−12·9	− 8·8	−4·1
31	−56·2	−56·2	0·0
June 4	−49·8	−55·8	+6·0
1891 April 24	+ 3·2	+ 4·6	−1·4
27	+47·9	+45·2	+2·7
May 3	−58·1	−63·2	+5·1

As an example of the delicacy of the spectrographic negatives, I remark that in those of *β Aurigæ* not only has the periodic doubling of the lines been perceptible and well measurable in the magnesium line $\lambda = 448\ \mu\mu$, but also on some plates in other very fine lines in its vicinity. These observations may be found in *Ast. Nach.*, No. 3017.

In order now to give an illustration of the accuracy which is

attainable by the method of covering Hγ by a strip, I add my observations on *α Lyræ* and *α Canis Majoris*:—

	Obs. Vel.	Earth's Vel.	Star's Vel. red. to ⊙.
	α Lyræ.		
1888 Sept. 28	− 2·6	+ 8·6	−11·2
Nov. 11	− 1·7	+ 7·0	− 8·7
13	− 1·5	+ 6·8	− 8·3
1889 May 31	−10·7	− 4·8	− 5·9
June 6	− 9·9	− 4·0	− 5·9
Sept. 15	− 1·8	+ 8·1	− 9·9
Nov. 24	− 4·2	+ 5·7	− 9·9
25	− 6·7	+ 5·6	−12·3
26	− 7·2	+ 5·4	−12·6
	α Canis Majoris.		
1888 Dec. 13	−14·1	− 4·9	− 9·2
1890 Feb. 12	+ 0·8	+ 9·4	− 8·6
1891 Feb. 7	+ 0·1	+ 8·3	− 8·2
Mar. 21	+ 4·4	+13·8	− 9·4
22	+ 6·1	+13·9	− 7·8

I remark, further, that the observations of *Sirius* by the method of stars for the second class give 7·3 miles, and with the aid of the iron comparison spectrum 9·0 miles as the rate of approach towards the Sun.

In regard to the exactness of the measurements in general I will state that out of the average of all the observations the resulting probable error of a single negative for stars of Class II. is ±1·34 mile; for the stars of Class I. ±2·31.

Each star has on the average been observed 3·3 times, and the measurements have been made independently by myself and by Dr. Scheiner. It may, therefore, be concluded that the probable error of the definitive values for both spectral classes will amount to less than one mile.

I intend after the definite completion of the measurements to communicate to the Society a list of the observed velocities, and will remark in conclusion that the velocities of the stars have proved to be much smaller than was to be expected from the direct observations. The mean result for forty-seven stars is 10·6 English miles.

Among them six have a velocity less than 2, and five greater than 20 miles; the greatest is that of *α Tauri*, about 30 miles. Fifteen of the stars have a positive and thirty-two a negative motion.

Potsdam, Royal Observatory:
1891 *December.*

The Motion of Σ 2525. By S. W. Burnham, M.A.

For some years this has been a difficult pair to measure. When it was discovered and first measured by Struve in 1830, it could be readily seen with the smallest instrument ordinarily used in double-star work. Since that time the distance has gradually lessened, and for the past few years only the largest refractors can separate the components. Until recently the change has been principally in distance, and all the measures made during the first fifty years are well represented by rectilinear motion. Notwithstanding that these positions appear to represent the movement of one star passing another from proper motion, the chances are greatly in favour of a physical relation between them.

I have made a set of measures of this pair with the 36-inch equatoreal under very favourable atmospheric conditions. Although the distance between the components is less than 0″·2, it is easy enough with this instrument, since it is well separated with medium powers, and the measures should have a good degree of accuracy.

The results found on the several nights are ollows:—

	°	″	
1891·326	343·7	0·20	8·0 . . . 8·1
·389	343·4	0·17	8·0 . . . 8·2
·427	163·7	0·17	8·0 . . . 8·5
·449	342·7	0·17	8·0 . . . 8·1
1891·40	343·4	0·18	

In very close pairs of this kind, and particularly when the relative motion is not well understood, it is important to assign the proper quadrant to the smaller star. In the first three measures the angles were set down as above with no thought of the earlier observations, with a note, "quadrant certain," in the second measure. On the following night the angle was reversed, and on the last night the smaller component seemed to be on the preceding side, although the difference in the magnitudes was very small. The magnitudes in Struve are 7·4 and 7·6.

In order to determine this question more certainly, if possible, I examined the pair again on several nights, with the following results:

1891·485. Not very good seeing, but the smaller star appears to be in 160°.

1891·540. The stars are so nearly equal that it is impossible to tell with certainty which is the smaller. Well separated.

1891·578. The preceding star appears to be certainly the smallest, but the difference is perhaps not more than 0·1 or 0·2 of a magnitude. Splendid seeing.

The quadrant, therefore, is still uncertain, but the weight of the evidence, from my own observations, seems to be in favour of the position-angle being as given in the mean result of the several measures, and not 163°·4; but the latter seems to be more in harmony with the earlier observations. This can probably be definitely settled by the observations of next year. If the angle is what I have assumed it to be, then the angular movement has been 270° since 1830, and we should have sufficient data for an approximate determination of the period. On the other hand, if this position belongs in the opposite quadrant, then all the measured positions lie substantially in a straight line, and it would be impossible to say from this evidence alone whether the relative change is due to proper or orbital motion.

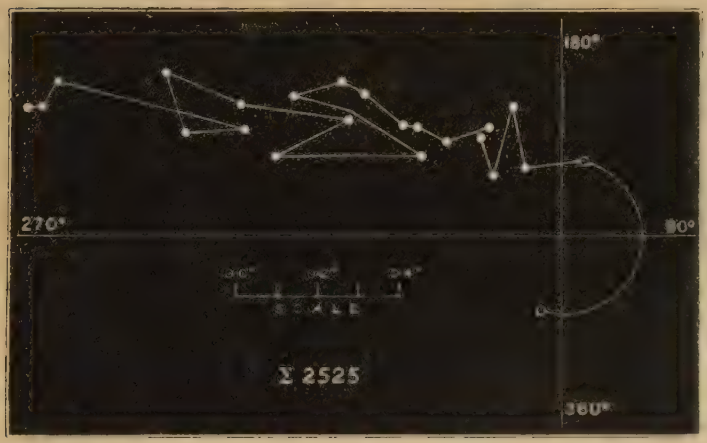
In the interest of a future investigation, I have collected from original sources, and arranged in chronological order, all the micrometrical observations of this pair, and give them here:—

Measures of Σ 2525.

		°	″		
1	1830·43	255·9	1·33	Σ.	5*n*
2	1836·14	255·5	1·30	Σ.	2*n*
3	1840·62	255·5	...	Da.	1*n*
	1840·70	252·4	1·28	OΣ.	2*n*
4	1842·41	251·0	0·82	Ma.	1*n*
5	1843·69	254·0	0·95	Ma.	1*n*
6	1854·63	246·8	1·05	OΣ.	1*n*
7	1856·61	247·1	0·85	Se.	2*n*
8	1865·22	240·9	0·60	Δ.	7*n*
9	1865·48	73·6	0·73	En.	2*n*
10	1865·64	239·9	0·40	Se.	1*n*
11	1865·76	241·5	0·74	OΣ.	2*n*
12	1872·61	234·0	0·66	OΣ.	1*n*
	1873·54	241·1	...	Ferr.	1*n*
13	1873·61	233·4	0·6	Δ	4*n*
	1873·65	232·1	...	WS.	3*n*
14	1874·84	234·3	0·48	Gl.	1*n*
15	1875·66	233·6	...	WS.	1*n*
	1875·65	232·1	0·45	Sp.	5*n*
16	1877·01	230·1	0·37	Sp.	5*n*
	1877·82	238·0	...	Δ	1*n*
17	1879·71	213·2	0·33	Hl.	3*n*
	1879·79	243·0	...	Do.	3*n*
18	1880·67	217·8	0·32	Sp.	6*n*

18	1880·71	210°·0	″...	Do.	2*n*
19	1883·26	47·9	0·23	En.	8*n*
20	1883·63	200·0	0·35	Sp.	4*n*
	1883·58	Elong. 27°		Per.	4*n*
21	1886·92	208·6	0·2 ±	Sp.	2*n*
	1887·71	Single		Sp.	4*n*
22	1891·40	343·4	0·18	*β*	4*n*

All of the measures in the foregoing list which are complete, twenty-two in number, are laid off to scale on the accompanying diagram as accurately as possible, and show at a glance all of the observed positions of this pair. The measures of 1891 are shown by outline circles in both quadrants. The successive measures are connected by lines, so that any one of them can be readily identified in the tabular list.



The resemblance between this pair and 20 *Draconis* is very striking, not only in the general direction of the components with reference to each other and their relative motions, but in the direction of the apparent movement of the respective companions. This will be very apparent by comparing the present diagram with that of 20 *Draconis* given in my paper on that pair in *Monthly Notices* for June 1891. The apparent relative motion of Σ 2525 is in the direction of about 83°, while that of 20 *Draconis*, as I have previously shown, is in the direction of 73°. Assuming that these are binary pairs—as is most probably the case—it is obvious that the apparent ellipses must be extremely elongated, and that the periods are of long duration. In a few years more,

doubtless, we shall have data for a better determination of these points.

The place of this star (1880) is:

R.A. $19^h\ 21^m\ 40^s$
Decl. $+27°\ 5'$

Mount Hamilton:
1891 *October* 31.

Note on some apparently variable Nebulæ.

By J. L. E. Dreyer, Ph.D.

In his ninth list of new nebulæ found at the Warner Observatory (*Astr. Nachr.*, No. 3004), Mr. Lewis Swift gave under No. 13 the following position (for 1890) and description of a nebula:

1889 December 23	$3^h\ 36^m\ 6^s$	$95°\ 2'{\cdot}1$	—	*ee*F, *p*L, R, 1st of 3 in line with 1417–18, cometary. Unable to refind it; seeing good. Failed also at Harvard College Observatory.

In No. 3014 of the same journal Mr. Swift remarks about this object: "I strongly suspect it to have been a comet, as at two subsequent examinations it could not be found. It was in line with N.G.C. 1417 and 1418, and all three were seen simultaneously."

From an examination of all the observations of nebulæ in the region in question, it appears to me to be more likely that we have here to do with a variable nebula and not with a comet. The following positions are all reduced to 1860:

	h	m	s	°	′		
				Sir William Herschel.			
III. 569	3	32	54	95	7		eF, *l*E, *er*
II. 455	3	34	59	95	8		Two, the *p* one *p*B, *c*L, E, *lb*M, the *f* one *e*F, *v*S, E.
II. 456	3	35	26	95	9		
				Sir John Herschel.			
h 305	3	34	32·5	95	7	±	No descr. 1st of 3 (one observation).
h 306	3	34	41·4	95	8	48	R, *np a* ✶, 2nd of 3 (one obs.).
h 307	3	35	1·8	95	9	36	*p*B, S, B ✶ 1′ *sf* (3 obs.).
				d'Arrest.			
III. 569	3	32	49·0	95	7	18	(2 obs.)
II. 455	3	35	0·5	95	9	28	(6 obs.)
II. 456	3	35	19·0	95	11	10	(3 obs.)

In the General Catalogue Sir John Herschel has only made use of his own observations, assuming III. 569 = *h* 305. That this was erroneous was pointed out by d'Arrest, whose positions of the three nebulæ agreed well with those determined by Sir W. Herschel. As I had also at Birr Castle in 1875–77 four times observed these three objects (and seen no others near them), and found their relative positions to be as indicated by d'Arrest, I adopted these in the New General Catalogue.

I had, however, when preparing the observations of nebulæ made with Lord Rosse's telescope for publication, been considerably puzzled at finding several observations from the years 1848–56, according to which there seemed to be a nebula nearly in the place of *h* 305. These observations are as follows:

1848 December 19.—3 neb. nearly in a line *nnp*.* Middle one the brightest, 2 others F. From 1st to 2nd about 10′, from 2nd to 3rd about 6′.

1850 October 7.—1st appears divided, and *p* part has a minute * or stellar point in it; 2nd about 11′ off is brightest of the 3 and *b*M; 3rd about 7′ from 2nd is equable in light, and I suspect a fourth F one perhaps 20′ off [G.C. 763].

1850 December 8.—1st is *v*F, and has a S**np*; 2nd has, I suspect, a F Nucl; it is *b*M, and the brightest of the 3, *n* of a *. About 16′ *f* the third is a F Nova [G.C. 763].

1851 January 22.—1st *ee*F, divided ? * in *p* part; 2nd 9′ *sff*, centre *r* ? Brightest of them. 3rd 4′ *sff* 2nd and 14′ *f* is one, *v*F, E.

1855 December 13.—Found only *h* 306 and 307 . . .

1856 December 26.—*p* one is *v*F, mottled; 2nd is *p*B, E*mp sf*, *b*MN; the 3rd is *v*F, *l*E ? no Nucl.

But in 1872, 1875, and three times in 1877 only *h* 306, *h* 307, and G.C. 763 were seen. The last observation (1877, December 6) I took for the special purpose of seeing whether there was a nebula 8′ or 10′ north preceding *h* 306, but as I could only see III. 569, 33′ preceding *h* 306, I concluded that the observers in 1848–56, who used Sir J. Herschel's Catalogue of 1833 as a working list, had been influenced by the relative positions in that Catalogue when estimating the distance between the first two nebulæ.

Now, however, the matter assumes a different aspect after Mr. Swift's observation. His position of the object seen in December 1889 is for 1860—

$3^h\ 34^m\ 37^s$ $95°\ 8'{\cdot}0$

or 6″·2 north preceding II. 455, with which and with II. 456 it was seen in a straight line. This position agrees well with that

* *nnp* should, no doubt, be *npp*.

determined by Sir John Herschel (*h* 305), as well as with the place given by the Birr Castle observers in 1848·56. This very faint nebula would seem to have been visible in 1827, 1848, 1850, 1851, 1856, and 1889, while it was not seen in 1785, 1855, 1864, 1865, 1872, 1875, 1877, and 1890. It is, however, important to remember that it was only *specially looked for* in 1877 and 1890, though d'Arrest probably also looked specially in the place given by Sir J. Herschel for *h* 305.

Observers of nebulæ are accustomed occasionally to find a particular object very difficult to see owing to atmospheric causes, but the present case is certainly a very suspicious one, and the region 6′ north preceding the nebula II. 455=N.G.C. 1417 should therefore be watched from time to time with powerful instruments. II. 455 and 456 are easily recognised, as they form a trapezium with two stars of 11th or 12th magnitude south following.

Some years ago Professor Winnecke drew attention to two nebulæ which he suspected of periodic variability, *h* 229 and *h* 882.* The former was invisible (or perhaps barely visible) to Schönfeld in December 1861 (6½-inch refractor), and invisible to Vogel on two nights in November 1865 (8½-inch refractor), while all other observers (including Schönfeld in 1864) called it *p*B, or of the second class. In November 1887 I noted it as "fully of the second class," and in December 1888 as "perhaps barely of the second class." The evidence of variability does not seem very strong in this case. As regards *h* 882, the variability seems scarcely better established. Great weight cannot be attached to the circumstance that Boguslawski (using a 3½-inch refractor) inserted it as bright on the Berlin map (*Hora XI.*), as he very possibly merely did so because it had been rated as a first-class nebula by W. Herschel. On May 24, 1887, I looked it up, but "could only just see it, and should certainly not have found it but for the star 6th mag. *nf*; twilight and strong haze." This observation is, however, of very little value, but on March 30, 1891, I found it "*p*B, though perhaps barely of the second class," and on April 8, 1891, "*p*B, good to observe" [with bright wire illumination]. This agrees with J. Herschel's, d'Arrest's, and Winnecke's estimates, and the suspicion of variability rests almost entirely on the fact that J. Herschel, in 1830, found it "*e*F, 2nd or 3rd class." It should be remembered that this is a diffused nebula with very slight central condensation, and, as far as my experience goes, the appearance of objects of this kind is far more influenced by the state of our atmosphere than that of nebulæ with a distinct condensation generally is.

In the notes to the General Catalogue (p. 32), Sir J. Herschel suggests that a change may have occurred in the nebula I 41=*h* 1452. "On April 5,† 1784, H. describes it as L, B, *r*,

* *Monthly Notices*, xxxviii. p. 104, *Astron. Nachr.*, No. 2293.

† Should be April 25 according to Caroline Herschel's copy of the sweeps in the Society's library.

*sb*M,* *i*R figure, Class I.; on March 3,† 1789, *p*B, *c*L, *i* Fig., *er*, many of the stars visible, so that it may be called a cluster." J. Herschel and d'Arrest estimated it as belonging to the third class. I observed it twice in May 1891, and found it "F, *p*L, diffused, perhaps double or with *v*F star involved south, very little or no condensation, irregular figure, barely second class." This is another case of an uncondensed nebula, and there does not seem any reason to think it variable. I may mention that the right ascension (according to *h*) adopted in the N. G. C. is correct, the nebula being 18^s following and $3'{\cdot}9$ north of the star *Munich* 8551.

In the *Astr. Nachr.*, No. 1520, d'Arrest announced that a small object stated by Schmidt to be 5^s preceding and $6'$ south of M 49 (*h* 1294) could not be seen by him, and that only two very small stars appeared in the place. As nothing has been seen in the place in question by any other observer, there seems no reason to assume that a nebula ever existed there. W. Herschel's II. 498 cannot have been the same as Schmidt's object, as H. would no doubt have pointed out that it was between II. 18 and M 49; and II. 498 is therefore beyond a doubt = II. 18.

Whatever view we take of the nature of nebulæ, the question whether these objects vary in brightness must be considered as one of paramount importance. It is for this reason that I have thought it desirable to call attention to Mr. Swift's nebula; but it is with some reluctance that I have done so, as the greatest possible caution is necessary in deciding whether any change has really taken place in objects of this class. The lessons taught us by the satellite of *Venus* and the intra-mercurial planets ought to be kept well in mind when we feel inclined to draw important conclusions from very uncertain observations, which may admit of a different interpretation.

Note on the Variability of Es-Birm 673 = D.M. + 39°·4208.
By the Rev. T. E. Espin, B.A.

This star is No. 118 of my list of "Some New Red and Orange-red Stars," *Monthly Notices*, xlvi. No. 5, p. 293. The brightness on the night of its discovery (1885 July 9) was 7·9, and it irregularly diminished to 9·0. Since this time it has been occasionally observed, and found to be practically invariable up to the present year. On comparing some of Dr. Wolf's photographs together I was struck by the differences in the magnitudes of this star. The colour is fine red, and consequently

* C. H. has "*lb*M," an important difference.
† C. H. has March 23.

the star is from two to three magnitudes fainter on the photographs. There are several small stars near, and on the night of November 19 their magnitudes were roughly estimated. The star is found on three of Dr. Wolf's photographs that he has kindly sent me. The third photograph is not dated, but, I believe, was taken in the beginning of November. The following are the magnitudes taken from the photographs :

1891 June 1	10·8
September 9 and 10	11·2
November	10·0

On November 19 the star was estimated as 7·6, and on November 26 as 7·8.

Tow Law, Darlington:
1891 December 9.

Note on the Stonyhurst Drawings of the Solar Spots and Faculæ.
By the Rev. Walter Sidgreaves.

At the commencement of the series of Sun-spot drawings, instituted by the late Fr. Perry in November 1880, it was decided to fill in the faculæ. so far as this could be done with certainty. No small difficulty was experienced in the attempt, for it seemed impossible to produce a faithful representation of them, and both the director and the observer were forced to be content with a skeleton tracing of the brighter parts which could be differentiated from the rest of the photosphere without chance of error. Experience, however, in the course of time, taught the observer the magic effect of motion imparted to a faint image; and as he slowly travelled the image of the Sun across the drawing-sheet, the patches stood out with a clearness of definition that excluded all doubt of the border-line between faculæ and photospheric glare. The method then adopted, and followed ever since, was first to outline the brighter parts upon the stationary image, and then to fill in the picture by sketching the fainter details taken from the image while moving it slowly to and fro across the paper. By this means a very trustworthy record was obtained; and it was much improved by adopting the suggestion, made by Sir G. Stokes in 1883, that the contrast of a red-lead tracing of the faculæ would greatly help the eye in its search through the drawings for the true relation between the dark spots and their glowing attendants. This is apparent in following the disturbances through all their changes, and in sifting their evidences for an answer to the query, Which is the forerunner of the other ?

During the entire period of Fr. Perry's direction of this observatory no clear instance of faculæ preceding the birth of a spot had been detected in the drawings. Faculæ were always most abundant after the birth of a spot, and always outlived it, lingering for weeks and sometimes for months before expiring. But, on the other hand, the drawings afford no positive evidence of the birth of a spot before the appearance of faculæ; while every spot of importance appears to have been attended from the beginning with at least a small surrounding of faculæ. So that, although it remains true that faculæ in no extensive form precede the birth of a spot, but develop and grow to maturity either along with the spot or after its decline, we must guard our conclusions against their extension to the absolute priority of the spot.

The chances of gaining the positive evidence about the priority are not favourable. The greater part of the Sun's surface, on which a spot may spring into life, offers no possibility of seeing the faculæ. And during the years of greater activity our chances are greatly reduced by the interlacing of old and new faculæ. It is only during the minimum period of spot-life, when the intervals are greater and old *débris* get cleared away before new storms begin, that we can well hope for the evidence we want.

The drawings of the recent minimum period of 1889 have been under careful study during the past twelve months, and we find amongst them two cases in which the evidence of first appearance is unquestionable. And both of these show faculæ before any trace of a spot appears.

On June 29 a small patch of faculæ was sketched near the eastern limb, in latitude $-40°\ 5$ and in longitude $252°$. There was no trace of a spot in the neighbourhood, and neither spot nor faculæ had been seen near the position for years. On the following day a small round spot appeared in latitude $-40°{\cdot}3$ and longitude $252°{\cdot}2$—*i.e.*, in the midst of the faculæ, the faculæ on this day being visible only just close round the spot. Again, on July 31, another small patch of faculæ appeared in latitude $-22°$, longitude $155°$, without any spot near it. It was seen again on the following day, and still without a spot. But on the third day, August 2, a spot was sketched in latitude $-21°{\cdot}9$, longitude $155°{\cdot}4$.

In both cases the faculæ were of small area, but bright. And there can be no doubt either of the faculæ or of the spots. Both were new. The faculæ were not remnants, and the spots were not revivals of old disturbances. We may not be able yet to conclude that faculæ are really forerunners of spots. The two spots referred to may have been for the time hidden from our view by the faculæ. But this has no appearance of probability, the faculæ being seen at a distance from the limb of quite one-tenth of the solar diameter. So far, therefore, as our drawings at these dates are witnesses to priority, their evidence stands for

some faculæ preceding the birth of a spot. And more of the same class of evidence may be found even in the years of greater spot frequency, when the records of their spots and faculæ have been more fully examined and the history of each group is more accurately written.

Stonyhurst Observatory,
Lancashire.

On the Determination of Azimuth by Elongations of Polaris.

By Harold Jacoby, B.A.

When Polaris has been observed within half an hour of elongation for the determination of azimuth, the "reduction to elongation" is frequently computed by the formulæ:

$$m = \frac{2 \sin^2 \frac{1}{2} (t_e - t)}{\sin 1''}$$

$$a_e - a = a_e \,.\, m \,.\, \sin 1''$$

where

t_e, a_e, are the hour angle and azimuth of *Polaris* at elongation,

t, a, „ „ „ „ the observation,

a_e and a being reckoned in seconds of arc, from the north point.

But we can avoid the use of the somewhat extended table for m by tabulating $a_e - a$ itself. For if we put

$$M = 6000 \sin 1'' \cdot m$$

$$f = \frac{a_e - 6000}{6000}$$

we shall have

$$a_e - a = M + fM \quad .$$

Table I. gives M, with the argument $t_e - t$; and Table II. gives f with the argument a_e. It will be seen that M is really an approximate value of $a_e - a$, and f a correction factor for M. This factor can be multiplied by M with Crelle's Tables, so that logarithms will not be required.

Table III. gives the values of a_e and t_e for all latitudes between 35° and 55°. It is intended to take the place of the logarithmic calculation of a_e and t_e, or at least to serve as a check. The columns headed A and T are computed with the declination 88° 45′ 0″, and furnish approximate values of a_e and t_e. The columns headed x and y give the variations, per second of δ, of the azimuth and hour angle. We shall have, therefore,

$$a_e = A + x(\delta - 88° 45')$$

$$t_e = T + y(\delta - 88° 45')$$

where $\delta - 88^\circ\ 45'$ is to be expressed in seconds of arc. These formulæ will give a_e and t_e with all necessary accuracy for many years.

The following observations, made by one of the students of the Summer School in Geodesy of Columbia College, will serve to illustrate the use of the tables. The data employed are—

Station: New London, New Hampshire, No. 3.
Date: 1891 July 13.
Longitude: 71° 59′ West.
Latitude: 43° 24′·0.
Polaris $\alpha = 1^h\ 18^m\ 49^s\cdot6$ $\qquad \delta = 88^\circ\ 43'\ 23''\cdot1$.

We have then

$$\delta - 88^\circ\ 45' = -96''\cdot9,$$

and, from Table III.,

for $\phi = 43^\circ\ 24'\cdot0$

	$A = 1^\circ\ 43'\ 13''\cdot89$,	$x = -1''\cdot3766$,	$T = 5^h\ 55^m\ 16^s\cdot3$,	$y = +{}^s\cdot0631$	
$x(\delta - 88^\circ\ 45')$	$= +2\ 13\cdot39$,	$y(\delta - 88^\circ\ 45')$	$=$	$-\ 6\cdot1$	
a_e	$= 1\ 45\ 27\cdot28$,	t_e	$= 5\ 55\ 10\cdot2$		
		α	$= 1\ 18\ 49\cdot6$		
Sidereal Time of Elongation			$= 19\ 23\ 39\cdot4$		

The values here obtained for a_e and t_e are the same as would be given by a logarithmic calculation. The observations of Polaris were (eastern elongation):

No.	Pos.	Sidereal Time.	Corrected Readings.	$t_e - t$	M. Table I.	fM Table II.	$a_e - a$	Reduced Readings.
		h m s	° ′ ″	m s	′	″	″	° ′ ″
1	D	18 56 1·3	39 37 22·1	27 38·1	43·57	2·40	45·97	39 38 8·1
2	...	18 59 18·8	27·5	24 20·6	33·83	1·86	35·69	38 3·2
3	...	19 9 11·4	41·0	14 28·0	11·95	0·66	12·61	37 53·6
4	...	19 11 20·8	56·0	12 18·6	8·67	0·48	9·15	38 5·2
5	R	19 20 26·3	219 37 43·9	3 13·1	0·59	0·03	0·62	219 37 44·5
6	...	19 23 11·7	38·5	0 27·7	0·01	0·00	0·01	38·5
7	...	19 27 24·4	32·1	3 45·0	0·81	0·04	0·85	33·0
8	...	19 29 17·7	29·2	5 38·3	1·81	0·10	1·91	31·1
9	...	19 33 49·5	33·7	10 10·1	5·90	0·32	6·22	39·9
10	...	19 37 4·0	18·6	13 24·6	10·28	0·57	10·85	29·4
11	...	19 41 0·6	16·8	17 21·2	17·19	0·95	18·14	34·9
12	...	19 45 0·3	7·3	21 20·9	26·02	1·43	27·45	34·8

The above observations are discordant, but they show how the tables are used.

TABLE I.

M.

m	0°	10°	20°	30°	40°	50°	Difference for 10°
	"	"	"	"	"	"	"
0	0·00	0·00	0·01	0·01	0·03	0·04	0·01
1	0·06	0·08	0·10	0·13	0·16	0·19	0·03
2	0·23	0·27	0·31	0·36	0·41	0·46	0·05
3	0·51	0·57	0·64	0·70	0·77	0·84	0·07
4	0·91	0·99	1·07	1·16	1·24	1·34	0·09
5	1·43	1·52	1·62	1·73	1·83	1·94	0·10
6	2·06	2·17	2·29	2·41	2·54	2·67	0·12
7	2·80	2·93	3·07	3·21	3·36	3·50	0·14
8	3·66	3·81	3·97	4·13	4·29	4·46	0·16
9	4·63	4·80	4·98	5·15	5·34	5·52	0·18
10	5·71	5·90	6·10	6·30	6·50	6·70	0·20
11	6·91	7·12	7·34	7·55	7·77	8·00	0·22
12	8·22	8·46	8·69	8·92	9·16	9·41	0·24
13	9·65	9·90	10·15	10·41	10·67	10·92	0·26
14	11·19	11·47	11·73	12·00	12·28	12·57	0·28
15	12·85	13·14	13·43	13·72	14·02	14·32	0·29
16	14·62	14·92	15·23	15·54	15·86	16·18	0·31
17	16·50	16·83	17·16	17·49	17·82	18·16	0·33
18	18·50	18·85	19·19	19·54	19·89	20·25	0·35
19	20·61	20·97	21·34	21·71	22·08	22·46	0·37
20	22·83	23·21	23·60	23·99	24·39	24·78	0·39
21	25·18	25·58	25·98	26·39	26·80	27·21	0·41
22	27·64	28·05	28·47	28·90	29·33	29·76	0·43
23	30·19	30·63	31·08	31·52	31·97	32·42	0·45
24	32·88	33·34	33·80	34·26	34·73	35·20	0·46
25	35·67	36·15	36·63	37·11	37·59	38·08	0·48
26	38·58	39·08	39·58	40·08	40·58	41·09	0·50
27	41·60	42·11	42·63	43·15	43·67	41·20	0·52
28	44·73	45·27	45·81	46·34	46·88	47·42	0·54
29	47·97	48·53	49·09	49·65	50·21	50·77	0·56

TABLE II.

f.

ae Az. at Elong.	*f.* Corr. Factor.	*ae* Az. at Elong.	*f* Corr. Factor.
° ′		° ′	
1 20	−0·200	2 0	+0·200
1 30	−0·100	2 10	+0·300
1 40	0·000	2 20	+0·400
1 50	+0·100		

TABLE III.

ϕ Latitude.	A Azimuth at Elongation.	x Var. of Azimuth for 1″ of δ.	T Hour Angle at Elongation.	y Var. of Hour Angle for 1″ of δ.
° ′	° ′ ″	″	h m s	s
35 0	1 31 33·70	−1·2210	5 56 29·9	+·0467
	11·24	24	1·3	3
10	1 31 44·94	−1·2234	5 56 28·6	+·0470
	11·33	26	1·3	3
20	1 31 56·27	−1·2260	5 56 27·3	+·0473
	11·43	25	1·3	3
30	1 32 7·70	−1·2285	5 56 26·0	+·0476
	11·52	26	1·3	3
40	1 32 19·22	−1·2311	5 56 24·7	+·0479
	11·61	25	1·4	3
50	1 32 30·83	−1·2336	5 56 23·3	+·0482
	11·71	26	1·3	3
36 0	1 32 42·54	−1·2362	5 56 22·0	+·0485
	11·81	27	1·3	3
10	1 32 54·35	−1·2389	5 56 20·7	+·0488
	11·90	26	1·4	3
20	1 33 6·25	−1·2415	5 56 19·3	+·0491
	12·01	27	1·4	3
30	1 33 18·26	−1·2442	5 56 17·9	+·0494
	12·10	27	1·3	3
40	1 33 30·36	−1·2469	5 56 16·6	+·0497
	12·20	27	1·3	3
50	1 33 42·56	−1·2496	5 56 15·3	+·0500
	12·31	27	1·4	3
37 0	1 33 54·87	−1·2523	5 56 13·9	+·0503
	12·40	28	1·4	3
10	1 34 7·27	−1·2551	5 56 12·5	+·0506
	12·51	28	1·4	3
20	1 34 19·78	−1·2579	5 56 11·1	+·0509
	12·61	28	1·4	3
30	1 34 32·39	−1·2607	5 56 9·7	+·0512
	12·73	28	1·4	3
40	1 34 45·12	−1·2635	5 56 8·3	+·0515
	12·81	28	1·3	3
50	1 35 57·93	−1·2663	5 56 7·0	+·0518
	12·93	29	1·4	3
38 0	1 35 10·86	−1·2692	5 56 5·6	+·0521
	13·04	29	1·5	3
10	1 35 23·90	−1·2721	5 56 4·1	+·0524
	13·14	29	1·4	3
20	1 35 37·04	−1·2750	5 56 2·7	+·0527
	13·25	30	1·4	4
30	1 35 50·29	−1·2780	5 56 1·3	+·0531
	13·37	29	1·4	3
40	1 36 3·66	−1·2809	5 55 59·9	+·0534
	13·47	31	1·4	3
50	1 36 17·13	−1·2840	5 55 58·5	+·0537
	13·59	30	1·5	3

Azimuth at Elongation $= A + x(\delta - 88° 45')$.
$= T + y(\delta - 88° 45')$.

Hour Angle at Elongation $(\delta - 88° 45')$ to be expressed in seconds of arc.

ϕ Latitude.	A Azimuth at Elongation.	x Var. of Azimuth for 1″ of δ.	T Hour Angle at Elongation.	y Var. of Hour Angle for 1″ of δ.
° ′	° ′ ″	″	h m s	s
39 0	1 36 30·72	−1·2870	5 55 57·0	+·0540
	13·70	30	1·5	4
10	1 36 44·42	−1·2900	5 55 55·5	+·0544
	13·82	31	1·4	3
20	1 36 58·24	−1·2931	5 55 54·1	+·0547
	13·92	31	1·4	3
30	1 37 12·16	−1·2962	5 55 52·7	+·0550
	14·05	31	1·5	3
40	1 37 26·21	−1·2993	5 55 51·2	+·0553
	14·17	32	1·5	3
50	1 37 40·38	−1·3025	5 55 49·7	+·0556
	14·28	32	1·5	4
40 0	1 37 54·66	−1·3057	5 55 48·2	+·0560
	14·40	32	1·5	3
10	1 38 9·06	−1·3089	5 55 46·7	+·0663
	14·53	32	1·5	3
20	1 38 23·59	−1·3121	5 55 45·2	+·0566
	14·65	32	1·5	4
30	1 38 38·24	−1·3153	5 55 43·7	+·0570
	14·76	33	1·5	3
40	1 38 53·00	−1·3186	5 55 42·2	+·0573
	14·90	33	1·5	3
50	1 39 7·90	−1·3219	5 55 40·7	+·0576
	15·02	34	1·6	4
41 0	1 39 22·92	−1·3253	5 55 39·1	+·0580
	15·14	33	1·5	3
10	1 39 38·06	−1·3286	5 55 37·6	+·0583
	15·28	34	1·5	4
20	1 39 53·34	−1·3320	5 55 36·1	+·0587
	15·40	34	1·6	3
30	1 40 8·74	−1·3354	5 55 34·5	+·0590
	15·53	35	1·6	4
40	1 40 24·27	−1·3389	5 55 32·9	+·0594
	15·67	35	1·5	3
50	1 40 39·94	−1·3424	5 55 31·4	+·0597
	15·80	35	1·6	4
42 0	1 40 55·74	−1·3459	5 55 29·8	+·0601
	15·93	36	1·6	3
10	1 41 11·67	−1·3495	5 55 28·2	+·0604
	16·07	35	1·5	4
20	1 41 27·74	−1·3530	5 55 26·7	+·0608
	16·20	36	1·6	3
30	1 41 43·94	−1·3566	5 55 25·1	+·0611
	16·35	37	1·7	4
40	1 42 0·29	−1·3603	5 55 23·4	+·0615
	16·48	36	1·6	3
50	1 42 16·77	−1·3639	5 55 21·8	+·0618
	16·63	37	1·6	4

Azimuth at Elongation $= A + x(\delta - 88° 45')$. Hour Angle at Elongation $= T + y(\delta - 88° 45')$. $(\delta - 88° 45')$ to be expressed in seconds of arc.

ϕ Latitude.	A Azimuth at Elongation.	x Var. of Azimuth for 1″ of δ.	T Hour Angle at Elongation.	y Var. of Hour Angle for 1″ of δ.
° ′	° ′ ″	″	h m s	s
43 0	1 42 33·40	−1·3676	5 55 20·2	+·0622
	16·77	37	1·7	4
10	1 42 50·17	−1·3713	5 55 18·5	+·0626
	16·91	38	1·6	3
20	1 43 7·08	−1·3751	5 55 16·9	+·0629
	17·06	38	1·6	4
30	1 43 24·14	−1·3789	5 55 15·3	+·0633
	17·20	38	1·7	4
40	1 43 41·34	−1·3827	5 55 13·6	+·0637
	17·36	39	1·7	3
50	1 43 58·70	−1·3866	5 55 11·9	+·0640
	17·50	39	1·7	4
44 0	1 44 16·20	−1·3905	5 55 10·2	+·0644
	17·65	39	1·7	4
10	1 44 33·85	−1·3944	5 55 8·5	+·0648
	17·81	40	1·7	4
20	1 44 51·66	−1·3984	5 55 6·8	+·0652
	17·97	40	1·7	4
30	1 45 9·63	−1·4024	5 55 5·1	+·0656
	18·12	40	1·7	4
40	1 45 27·75	−1·4064	5 55 3·4	+·0660
	18·28	41	1·7	3
50	1 45 46·03	−1·4105	5 55 1·7	+·0663
	18·44	41	1·8	4
45 0	1 46 4·47	−1·4146	5 54 59·9	+·0667
	18·60	41	1·7	4
10	1 46 23·07	−1·4187	5 54 58·2	+·0671
	18·76	42	1·8	4
20	1 46 41·83	−1·4229	5 54 56·4	+·0675
	18·93	42	1·7	4
30	1 47 0·76	−1·4271	5 54 54·7	+·0679
	19·10	42	1·8	4
40	1 47 19·86	−1·4313	5 54 52·9	+·0683
	19·26	43	1·8	4
50	1 47 39·12	−1·4356	5 54 51·1	+·0687
	19·44	43	1·8	4
46 0	1 47 58·56	−1·4399	5 54 49·3	+·0691
	19·60	44	1·8	4
10	1 48 18·16	−1·4443	5 54 47·5	+·0695
	19·79	44	1·9	4
20	1 48 37·95	−1·4487	5 54 45·6	+·0699
	19·95	44	1·8	4
30	1 48 57·90	−1·4531	5 54 43·8	+·0703
	20·14	45	1·9	4
40	1 49 18·04	−1·4576	5 54 41·9	+·0707
	20·32	45	1·8	4
50	1 49 38·36	−1·4621	5 54 40·1	+·0711
	20·50	46	1·9	4

Azimuth at Elongation $= A + x(\delta - 88° 45')$. Hour Angle at Elongation $= T + y(\delta - 88° 45')$. $(\delta - 88° 45')$ to be expressed in seconds of arc.

ϕ Latitude.	A Azimuth at Elongation.	x Var. of Azimuth for 1″ of δ.	T Hour Angle at Elongation.	y Var. of Hour Angle for 1″ of δ.
° ′	° ′ ″	″	h m s	s
47 0	1 49 58·86	−1·4667	5 54 38·2	+·0715
	20·68	46	1·9	5
10	1 50 19·54	−1·4713	5 54 36·3	+·0720
	20·87	46	1·9	4
20	1 50 40·41	−1·4759	5 54 34·4	+·0724
	21·06	47	1·9	4
30	1 51 1·47	−1·4806	5 54 32·5	+·0728
	21·25	48	1·9	4
40	1 51 22·72	−1·4854	5 54 30·6	+·0732
	21·45	47	1·9	4
50	1 51 44·17	−1·4901	5 54 28·7	+·0736
	21·63	48	2·0	5
48 0	1 52 5·80	−1·4949	5 54 26·7	+·0741
	21·84	49	1·9	4
10	1 52 27·64	−1·4998	5 54 24·8	+·0745
	22·03	49	2·0	5
20	1 52 49·67	−1·5047	5 54 22·8	+·0750
	22·24	49	2·0	4
30	1 53 11·91	−1·5096	5 54 20·8	+·0754
	22·44	50	2·0	4
40	1 53 34·35	−1·5146	5 54 18·8	+·0758
	22·65	51	2·0	5
50	1 53 57·00	−1·5197	5 54 16·8	+·0763
	22·86	51	2·0	4
49 0	1 54 19·86	−1·5248	5 54 14·8	+·0767
	23·07	51	2·1	5
10	1 54 42·93	−1·5299	5 54 12·7	+·0772
	23·28	52	2·0	4
20	1 55 6·21	−1·5351	5 54 10·7	+·0776
	23·51	52	2·1	5
30	1 55 29·72	−1·5403	5 54 8·6	+·0781
	23·71	53	2·1	5
40	1 55 53·43	−1·5456	5 54 6·5	+·0786
	23·95	53	2·0	4
50	1 56 17·38	−1·5509	5 54 4·5	+·0790
	24·17	54	2·1	5
50 0	1 56 41·55	−1·5563	5 54 2·4	+·0795
	24·39	54	2·1	5
10	1 57 5·94	−1·5617	5 54 0·3	+·0800
	24·63	55	2·2	4
20	1 57 30·57	−1·5672	5 53 58·1	+·0804
	24·86	55	2·2	5
30	1 57 55·43	−1·5727	5 53 55·9	+·0809
	25·09	56	2·1	5
40	1 58 20·52	−1·5783	5 53 53·8	+·0814
	25·34	56	2·2	5
50	1 58 45·86	−1·5839	5 53 51·6	+·0819
	25·58	57	2·2	5

Azimuth at Elongation $= A + x(\delta - 88° 45')$. Hour Angle at Elongation $= T + y(\delta - 88° 45')$. $(\delta - 88° 45')$ to be expressed in seconds of arc.

ϕ Latitude.	A Azimuth at Elongation.	x Var. of Azimuth for 1″ of δ.	T Hour Angle at Elongation.	y Var. of Hour Angle for 1″ of δ.
° ′	° ′ ″	″	h m s	s
51 0	1 59 11·44	−1·5896	5 53 49·4	+·0824
	25·83	57	2·2	5
10	1 59 37·27	−1·5953	5 53 47·2	+·0829
	26·06	58	2·2	5
20	2 0 3·33	−1·6011	5 53 45·0	+·0834
	26·32	59	2·3	5
30	2 0 29·65	−1·6070	5 53 42·7	+·0839
	26·58	59	2·2	5
40	2 0 56·23	−1·6129	5 53 40·5	+·0844
	26·83	60	2·3	5
50	2 1 23·06	−1·6189	5 53 38·2	+·0849
	27·10	60	2·3	5
52 0	2 1 50·16	−1·6249	5 53 35·9	+·0854
	27·36	61	2·3	5
10	2 2 17·52	−1·6310	5 53 33·6	+·0859
	27·63	62	2·3	5
20	2 2 45·15	−1·6372	5 53 31·3	+·0864
	27·90	62	2·4	6
30	2 3 13·05	−1·6434	5 53 28·9	+·0870
	28·18	63	2·4	5
40	2 3 41·23	−1·6497	5 53 26·5	+·0875
	28·46	63	2·4	5
50	2 4 9·69	−1·6560	5 53 24·1	+·0880
	28·74	64	2·4	6
53 0	2 4 38·43	−1·6624	5 53 21·7	+·0886
	29·02	64	2·4	5
10	2 5 7·45	−1·6688	5 53 19·3	+·0891
	29·32	65	2·4	5
20	2 5 36·77	−1·6753	5 53 16·9	+·0896
	29·60	66	2·4	6
30	2 6 6·37	−1·6819	5 53 14·5	+·0902
	29·91	67	2·5	5
40	2 6 36·28	−1·6886	5 53 12·0	+·0907
	30·21	67	2·5	6
50	2 7 6·49	−1·6953	5 53 9·5	+·0913
	30·52	68	2·6	5
54 0	2 7 37·01	−1·7021	5 53 6·9	+·0918
	30·82	69	2·5	6
10	2 8 7·83	−1·7090	5 53 4·4	+·0924
	31·15	69	2·5	
20	2 8 38·98	−1·7159	5 53 1·9	+·0930
	31·46	70	2·6	5
30	2 9 10·44	−1·7229	5 52 59·3	+·0935
	31·78	70	2·6	6
40	2 9 42·22	−1·7299	5 52 56·7	+·0941
	32·11	72	2·6	6
50	2 10 14·33	−1·7371	5 52 54·1	+·0947
	32·45	74	2·7	6
55 0	2 10 46·78	−1·7443	5 52 51·4	+·0953

Azimuth at Elongation $= A + x(\delta - 88° 45')$. Hour Angle at Elongation $= T + y(\delta - 88° 45')$. $(\delta - 88° 45')$ to be expressed in seconds of arc.

Columbia College, New York,
1891 *September* 30.

On the Reduction of Transit Observations by the Method of Least Squares. By Harold Jacoby, B.A.

When it is desired to adjust a series of observations by the method of least squares, the process usually employed is not always the most advantageous one available—in fact, the usual method is really an ingenious way of finding the *adjusted* values of the unknowns from the coefficients and absolute terms of the original observation equations. Occasionally, however, these coefficients are themselves all functions of some single quantity. In such a case there must exist a functional relation between the adjusted values of the unknowns and the single quantity in question; and this relation may possibly be evaluated *without computing the actual coefficients of the observation equations.* The above applies to the transit instrument in the meridian, for the coefficients of the observation equations, as well as the weight-factor, are then all functions of the declination. The *latitude* affects them as a constant only.

Before going into the details of the application of the above remarks to actual work, it is desirable to call attention to a theorem in the method of least squares which is very often of considerable use. Let there be given two groups of linear observation equations of the form:

$$\begin{aligned} a_1w + b_1y + c_1z + d_1 &= 0, & a_1'x + b_1'y + c_1'z + d_1' &= 0, \\ a_2w + b_2y + c_2z + d_2 &= 0, & a_2'x + b_2'y + c_2'z + d_2' &= 0, \\ \cdots\cdots\cdots & & \cdots\cdots\cdots & \end{aligned}$$

in which it will be seen that the equations of the first group lack the term in x, and those of the second group lack the term in w. Now let normal equations be formed from each group as follows:

$$\begin{aligned} [aa]w + [ab]y + [ac]z + [ad] &= 0, & [a'a']x + [a'b']y + [a'c']z + [a'd'] &= 0, \\ [bb]y + [bc]z + [bd] &= 0, & [b'b']y + [b'c']z + [b'd'] &= 0, \\ [cc]z + [cd] &= 0, & [c'c']z + [c'd'] &= 0, \\ [dd] & & [d'd']. & \end{aligned}$$

Let w and x be eliminated in the usual way, giving:

$$\begin{aligned} [bb\,.\,1]y + [bc\,.\,1]z + [bd\,.\,1] &= 0, & [b'b'\,.\,1]y + [b'c'\,.\,1]z + [b'd'\,.\,1] &= 0, \\ [cc\,.\,1]z + [cd\,.\,1] &= 0, & [c'c'\,.\,1]z + [c'd'\,.\,1] &= 0, \\ [dd\,.\,1] & & [d'd'\,.\,1]. & \end{aligned}$$

Now let these equations be added, term by term, giving:

$$\begin{aligned} ([bb\,.\,1] + [b'b'\,.\,1])y + ([bc\,.\,1] + [b'c'\,.\,1])z + ([bd\,.\,1] + [b'd'\,.\,1]) &= 0, \\ ([cc\,.\,1] + [c'c'\,.\,1])z + ([cd\,.\,1] + [c'd'\,.\,1]) &= 0, \\ ([dd\,.\,1] + [d'd'\,.\,1]); & \end{aligned}$$

then the equations last obtained are *identically* the same as would have resulted if we had formed a set of normal equations from the two original groups of observation equations, taken altogether as a single group, and then eliminated w and x according to the usual rule.*

When transit observations are reduced upon the assumption that the azimuth changed when the instrument was reversed,† we have two groups of observation equations of the kind just mentioned, and may take advantage of the above principle. It is, perhaps, out of place here to remark upon the desirability of making such an assumption, since the method to be given is equally applicable in any case. Perhaps the strongest argument against the use of a least square solution, upon the hypothesis of two values of the azimuth, is that the actual observation times of the reversal are in no way affected by the azimuth of the instrument before reversal, and ought therefore to have no influence in the determination of that quantity. It is certain, however, that they do obtain such an influence, if a least square adjustment is made upon the assumption of two azimuths. Yet the same objection will apply to observations of any kind in which some of the observation equations have one or more zero coefficients, a case occurring very frequently in practice. We have abundant authority for applying the method of least squares in such cases. It only remains to inquire whether the hypothesis of a change of azimuth is warranted by facts. Does such a change really occur? Upon this point no decisive evidence has been published, so far as I know. It must, therefore, be left to each observer to decide whether he has sufficient skill to reverse his instrument with a certainty of not appreciably changing the azimuth adjustment. If so, he would violate a known fact, by assuming that the azimuth changed in reversing. It seems, however, that in the majority of cases it is quite safe to assume two azimuths. It will be shown below that it is possible to make separate reductions on *both* hypotheses without very extensive extra calculations.

So much being premised, we can proceed to explain the proposed method of reduction, beginning with the process applicable to the hypothesis of *no change of azimuth*. If we assume the usual notation, we shall have, by Mayer's formula, for each observation a weighted equation of the form:

$$\sqrt{p}\,Ax \pm \sqrt{p}\,Cy + \sqrt{p}\,Bz + \sqrt{p}\,l = 0 \begin{cases} \text{Clamp West.} \\ \text{Clamp East.} \end{cases}$$

* For a demonstration of this interesting theorem, see *Jordan, Handbuch der Vermessungskunde, erster Band (Ausgleichungsrechnung) dritte Auflage*, 1888, p. 78.

† Chauvenet, *Spherical and Practical Astronomy*, vol. ii., p. 201. Doolittle, *Practical Astronomy*, p. 325.

in which, as usual:

$$A=\frac{\sin(\phi-\delta)}{\cos\delta},\ B=\frac{\cos(\phi-\delta)}{\cos\delta},\ C=\frac{1}{\cos\delta},\ p=\text{weight},$$

$$l=T-\alpha+\Delta\theta_0+Aa_0+Bb\pm Cc_0-0^s.021\cos\phi\sec\delta\begin{cases}\text{Clamp West},\\ \text{Clamp East},\end{cases}$$

and $\Delta\theta_0$, a_0, and c_0 are approximate values of the chronometer correction, azimuth constant, and collimation constant.* After the solution has been carried out, we have as the adjusted values—

$$a=a_0+x,\ c=c_0+y,\ \Delta\theta=\Delta\theta_0+z.$$

Now let us write—

$$m_1=\pm p\sec\delta,\ m_2=\pm p\tan\delta\sec\delta,\ m_3=p,\ m_4=p\tan\delta,\ m_5=p\tan^2\delta,$$

$$m_6=p\sec^2\delta\begin{cases}\text{Clamp West},\\ \text{Clamp East},\end{cases}$$

and put

$$B'=\sin\phi\,[m_1]-\cos\phi\,[m_2],$$
$$C'=\sin\phi\,[m_3]-\cos\phi\,[m_4],$$
$$L=\sin\phi\,[m_3l]-\cos\phi\,[m_4l],$$
$$K=\sin^2\phi\,[m_3]+\cos^2\phi\,[m_5]-2\sin\phi\cos\phi\,[m_4].$$

We do not then need to form the usual normal equations, which would be

$$\begin{aligned}[pAA]z+[pAC]y+[pA]z+[pAl]&=0,\\ [pCC]y+[pC]z+[pCl]&=0,\\ [p]z+[pl]&=0,\\ [pll],&\end{aligned}\qquad(a)$$

for we can write at once the *reduced* normal equations

$$\begin{aligned}[pCC\,.\,1]y+[pC\,.\,1]z+[pCC\,.\,1]&=0,\\ [p\,.\,1]z+[pl\,.\,1]&=0,\\ [pll\,.\,1],&\end{aligned}\qquad(b)$$

by means of the relations

$$[pCC\,.\,1]=[m_6]-\frac{B'^2}{K},\ [pC\,.\,1]=[m_4]-\frac{B'C'}{K},\ [pCl\,.\,1]=[m_4l]-\frac{B'L}{K},$$

$$[p\,.\,1]=[m_3]-\frac{C'^2}{K},\ [pl\,.\,1]=[m_3l]-\frac{C'L}{K},$$

$$[pll\,.\,1]=[m_3ll]-\frac{L^2}{K}.$$

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* In this investigation, the collimation constant is considered positive when stars at upper transit require a positive correction for clamp West. It should also be noticed that when the instrument is *very* well adjusted, a_0 and c_0 can be put equal to 0, whereby the computation of l is much facilitated.

From the equations (*b*), now involving only two unknowns, we get the adjusted values y and z in the usual way. If we wish the value of x also, we get it from the equation—

$$x = -\frac{B'}{K}y - \frac{C'}{K}z - \frac{L}{K}, \qquad (c)$$

where all the coefficients in the second member are known. It will be noticed that the quantities B′, C′, L, K are really coefficients of the usual normal equations (*a*), but by proceeding at once to the equations (*b*), we point out the method of reduction for the case of *two* different azimuth constants. We have then, in accordance with the principle already explained, to proceed by the following rule:—

Carry out the reduction for the clamp West and clamp East observations separately, until two separate sets of equations of the form (*b*) are obtained. Add corresponding coefficients in the two sets, and solve the resulting set in the usual way.

The values of y and z having thus been obtained, the two values of x for clamp West and clamp East result from an application of (*c*) to the clamp West and clamp East observations respectively. It is, of course, obvious that separate values of a_0 may be assumed in the beginning for the clamp West and clamp East observations, and separate values of B′, C′, L, K will also be found. The peculiar advantage of this method arises from the possibility of tabulating the quantities m_1, m_2, m_3, m_4, m_5, m_6, in tables of single entry. As they are functions of δ only, the values given in the tables which I have computed are good for any latitude. In computing the tables Safford's weight formula* was used. This gives

$$p = \frac{1{\cdot}3}{1 + 0{\cdot}3 \sec^2\delta},$$

which is often intermediate between the values given by other well-known formulas.† After all, we need only an approximate value of the weight; and, if necessary, tables to agree with any other weight formula could easily be computed from mine by simply multiplying each number by the ratio of the two weights at that particular declination.

It is evident that after an adjustment has been made upon the hypothesis of two azimuths, it is quite easy to obtain the values that would have resulted from the hypothesis of an unchanged azimuth; and such a double solution applied to a number of cases would throw much light upon the question of the desirability of the assumption of two azimuths.

[The tables referred to have been communicated by the author in manuscript. They are preserved in the archives of the Society.]

* *Transactions of the Wisconsin Academy of Science*, vol. vii., 1883–87, p. 199.

† Young, *Washington Observations*, 1878, Appendix ii., p. 29.

Observation of Comet b 1891 (*Wolf*), *made at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer-Royal.*)

The observation was made with the East or Sheepshanks Equatoreal, aperture 6·7 inches, by taking transits over two cross-wires at right angles to each other, and each inclined 45° to the parallel of declination. Magnifying power 55.

Greenwich Mean Solar Time.	Observer.	☄ − ★ R.A.	Corr. for Parallax.	Corr. for Refraction.	☄ − ★ N.P.D.	Corr. for Parallax.	Corr. for Ref.	No. of Comps.	Apparent R.A.	Tabular R.A.	Apparent N.P.D.	Tabular N.P.D.
d h m s		s	s	s	′ ″	″	″		h m s	h m s	° ′ ″	° ′
Dec. 2 10 10 41	A.C.	−29·90	−·15	+·05	+15 6·1	−8·5	+2·7	5	4 23 36·3	4 23 37	103 32 9·7	103 32·0

Assumed Mean Place of Comparison Star.

Name and Authority.	R.A. 1891·0.	N.P.D. 1891·0.
	h m s	° ′ ″
Greenwich 10-Year Catalogue 1880, No. 718	4 24 3·15	103 17 21·1

In computing the parallax log Δ has been assumed 9·9703, this being interpolated from Berberich's Ephemeris, as is also the Tabular Place. Here and in the observations communicated in the *Monthly Notices* for November, a second term has been applied in the N.P.D. Corr. for Refraction, depending on the difference between apparent and true orientation of the wires; this point was discussed by Col. Tupman in vol. xlviii., p. 96, of the *Monthly Notices*.

The initials "A.C." are those of Mr. Crommelin.

Observations of Comet Encke, made at the Radcliffe Observatory, Oxford.

(*Communicated by E. J. Stone, M.A., F.R.S., Radcliffe Observer.*)

The following observations were made by Mr. Robinson with the Barclay Equatoreal, using the Ring Micrometer, and a power of 100.

Date.	G.M.T.	Local Sidereal Time.	Observer.	Comet *minus* Star. (Corrected for Refraction only). R.A.	N.P.D.	No. of Comps.	Apparent R.A. of Comet.	Parallax in R.A. p	Log. $(p \times \Delta)$	Apparent N.P.D. of Comet.	Parallax in N.P.D. q	Log. $(q \times \Delta)$	Reference to Comp. Star.
1891.	h m s	h m s		m s	′ ″		h m s	s		° ′ ″	″		
Aug. 12	14 10 7	23 30 0	R.	+1 0·48	+3 10·69	9	4 37 46·92	−0·31	9·6254	57 23 31·9	−3·8	0·7116	(*a*)
Sept. 10	14 21 4	1 35 19	R.	+0 16·98	+7 8·84	9	7 49 59·08	−0·46	9·6382	56 57 2·0	−6·3	0·7765	(*b*)
	14 46 40	2 0 59	R.	−0 27·64	−5 39·44	6	7 50 8·67	−0·46	9·6378	56 57 30·5	−5·9	0·7519	(*c*)
30	16 40 8	5 13 37	R.	−0 45·67	+2 44·00	3	10 41 13·30	−0·39	9·5753	73 58 13·9	−6·6	0·8077	(*d*)
	16 56 24	5 29 55	R.	−0 23·57	+1 53·03	14	10 41 18·72	−0·38	9·5698	73 59 5·9	−6·5	0·8006	(*e*)
Oct. 4	16 27 27	5 16 40	R.	−1 32·60	−1 51·15	2	11 11 23·19	−0·37	9·5707	78 46 29·3	−6·6	0·8291	(*f*)
	16 27 27	5 16 40	R.	−1 48·12	−2 1·37	2	11 11 22·81	−0·37	9·5707	78 46 26·9	−6·6	0·8291	(*g*)
11	17 27 22	6 44 21	R.	−1 15·27	+9 26 93	1	12 2 51·19	−0·32	9·5556	87 38 53·9	−6·0	0·8362	(*h*)

Assumed Places of Comparison Stars.

Comp. Star.	Mean R.A. 1891·0.	Reduction to App. R.A.	Mean N.P.D. 1891·0.	Reduction to App. N.P.D.	Authority.
	h m s	s	° ′ ″	″	
(*a*)	4 36 45·76	+0·68	57 20 23·3	−2·1	Mean of W.B. (2) IV. 750 and Grant 1138.
(*b*)	7 49 41·41	+0·69	56 49 52·9	+0·3	Radcliffe Transit-circle observation, 1891, Nov. 22.
(*c*)	7 50 35·62	+0·69	57 3 9·6	+0·3	Leiden A.G., Zone 259, No. 118.
(*d*)	10 41 58·57	+0·40	73 55 28·5	+1·5	Radcliffe Transit-circle observation, 1891, Nov. 22.
(*e*)	10 41 41·89	+0·40	73 57 11·4	+1·5	Radcliffe Transit-circle observation, 1891, Nov. 22.
(*f*)	11 12 55·83	+0·34	78 48 18·8	+1·6	Mean of Brussels 4659, and Grant 2914.
(*g*)	11 13 10·92	+0 34	78 48 26·7	+1·6	Mean of Brussels 4661, and Grant 2916.
(*h*)	12 4 6·17	+0 29	87 29 24·8	+2·2	Greenwich (1880), 1892.

Observer's Remarks.

1891, August 12.—The Comet's light is of the feeblest, requiring averted vision to render it visible. No nucleus seen. Centre of patch observed. Sky brilliantly clear.

September 10.—The Coma is fan-shaped, thus:—

The nucleus, which is just within the obtuse angle on the west side, is mag. 13·5.

September 30.—The nucleus is now fairly bright (say 11 mag.). The Coma is very similar in form to that of September 10, but strong twilight obliterated the fainter parts of the Comet.

October 4.—Comet and stars faint; altitude small; sky hazy. Observations made with considerable difficulty.

October 11.—Comet just visible in very strong twilight when stars below the 7th magnitude had disappeared from the field of view.

In the computation of the parallaxes the value 8″·85 has been adopted for the Sun's mean horizontal parallax, and the geocentric distances, Δ, have been taken from an ephemeris by Dr. O. Backlund, given in the *Astronomische Nachrichten*, No. 3048.

Radcliffe Observatory, Oxford:
1891 *December* 3.

Observations of Comet Barnard 1891, *made at Sydney Observatory.*

(*Communicated by H. C. Russell, B.A., F.R.S., Government Astronomer.*)

This comet was found on October 9. It is a faint, round body with slight central condensation. The observations were made with the Filar Micrometer on the 11½-inch equatoreal, Mr. R. P. Sellars observing. Cloudy weather and then moonlight have prevented us from obtaining any observations since the 11th.

1891.	Syd. M. T.	Δ R.A.	Δ N.P.D.	cp.	RA. app.	log p. Δ	N.P.D. app.	log p. Δ	Red. ad l. app.		*
	h m s	m s	′ ″		h m s		° ′ ″		s	″	
Oct. 9	13 52 24	+0 58·31	− 4 34·52	22	8 9 23·26	9·773*n*	126 34 52·53	0·462	+0·50	−16·71	1
9	16 2 51	−0 0·61	+ 5 17·99	15	8 10 0·09	9·626*n*	126 42 18·32	9·877	+0·51	−16·61	2
11	13 58 58	+5 49·39	− 1 12·44	12	8 23 17·40	9·789*n*	129 14 56·96	0·428	+0·45	−16·60	3
11	15 45 12	+2 17·42	−10 1·17	13	8 23 48·70	9·683*n*	129 20 49·14	9·887	+0·41	−16·36	4

Mean Places of Comparison Stars for 1891·0.

*	RA. 1891·0.	N.P.D. 1891·0.	Authority.
	h m s	° ′ ″	
1	8 8 24·45	126 39 43·76	St. 4204, Arg. Gen. Cat. 10931 and 10964 (1^m in error).
2	8 10 0·19	126 37 16·94	Arg. Gen. Cat. 10980.
3	8 17 27·56	129 16 26·00	St. 4297, Arg. Gen. Cat. 11199.
4	8 21 30·87	129 31 6·67	Arg. Gen. Cat. 11311.

Sydney Observatory: 1891 *October* 19.